A LINEAR STABILITY ANALYSIS FOR CURVE SQUEALING OF TRAINS

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ABSTRACT

Curve squealing of railway wheels occurs in narrow curves with a frequency of about 4 kHz and is caused by a self-excited stick-slip oscillation in the wheel-rail contact. A squeal model, consisting of the first modal forms of an elastic wheel and a three-dimensional hard Coulomb contact, is developed to analyze the stability of the stationary run through a curve via linearization about the steady sliding equilibrium. The results show that in particular the front inner of the four wheels of a certain bogie tends to squeal.

INTRODUCTION

Curve squealing of railway wheels erratically occurs in narrow curves with a frequency of about 4000 Hz. In order to numerically analyze the squeal phenomenon, a mechanical model tailored for resolving frequencies up to 6 kHz has been developed in [1], on which the linear stability analysis has been performed. This model consists of a single non-rotating elastic wheel with inertially fixed center, and a rigid rail which moves with constant speed relative to the wheel according to the nominal creep that is present in curves. The interaction between the wheel and the rail is taken into account by one frictional contact point, for which a hard unilateral constraint is used in the normal direction, and a Coulomb friction element with constant friction coefficient in the two tangential directions. The elastic modes of the wheel that are used in this squeal model have been extracted in a previous step from FE-computations with succeeding modal reduction of a real railway wheel. In addition to the elastic modes of the wheel, the squeal model requires as inputs the location of the contact points, the inclination of the contact plane, the direction and magnitude of the nominal creep, and approximates of the nominal contact forces as they appear during a stable stationary run through a curve. In order to obtain these data, the stationary run of a specific driving trailer through a specific curve has been simulated with commercial multibody software and has been supplemented with measurements.

LINEAR STABILITY UNDER SLIDING FRICTION

The Newton-Euler equations for the impact-free motion of the squeal model have the form

\[ M \ddot{q} + D \dot{q} + Kq - w_N \lambda_N - W_T \lambda_T = 0 \]  

with \( q(t) \) being the modal amplitudes, \( M, D, \) and \( K \) the constant and symmetric pd. mass, modal damping, and stiffness matrix, \( (\lambda_N, \lambda_T) \) the scalar values of the normal and tangential contact forces, and \( (w_N, W_T) \) the three constant generalized force directions associated with them. Contact kinematics is described by

\[ g_N = w_N^T \dot{q} + a, \quad \gamma_N(\dot{q}) = w_N^T \dot{q}, \quad \gamma_T(q) = W_T^T \dot{q} + \chi_T, \]

where \( g_N \geq 0 \) is the gap function, and \( a \) a constant geometry parameter that takes into account the distance between the wheel’s
center and the contact point. The contact relative velocities in the normal and tangential directions are denoted by $\langle \gamma_N, \gamma_T \rangle$, and the nominal creep velocity by $\chi_T$. The latter acts as a kinematic excitation in the tangential directions. The standard contact laws for a closed contact are

$$g_N = 0 \Rightarrow \gamma_N \in \mathcal{N}_{B_2^N}(-\lambda_N), \quad \gamma_T \in \mathcal{N}_{\mu\lambda_N B_2}(-\lambda_T),$$  

where $\mathcal{N}$ denotes the normal cone, $\mu$ is the friction coefficient, and $B_2$ the two-dimensional unit ball. For the linear stability analysis we follow the general approach in [2] applied to the particular case of a steady sliding equilibrium under enduring contact, $(q_0, \lambda_{N0})$. Resolving for this case the contact laws (3) yields, together with (2), the Newton-Euler equations (1) in the form

$$M\ddot{q} + D\dot{q} + Kq - (w_N - W_T\mu e_T(q))\lambda_N = 0$$  

with $e_T(q) := \gamma_T(q)/||\gamma_T(q)||$ being the sliding direction. Within the framework of linear stability analysis, we assume $(q(t), \lambda_N(t))$ in (4) to be composed of the equilibrium $(q_0, \lambda_{N0})$ and small superimposed deviations $(\gamma(t), \nu_N(t))$,

$$q(t) = q_0 + \gamma(t), \quad \lambda_N(t) = \lambda_{N0} + \nu_N(t).$$  

We further denote by $e_T(0) := e_T(0)$ the nominal creep direction. By a Taylor series expansion of (4), one obtains together with (2) the conditions

$$Kq_0 - (w_N - W_T\mu e_T(0))\lambda_{N0} = 0$$
$$g_{N0} = w_N^T q_0 + a = 0$$  

for the steady sliding equilibrium $(q_0, \lambda_{N0})$, as well as the first order approximation of the wheel’s dynamics

$$M\ddot{\gamma} + D\dot{\gamma} + K\gamma - (w_N - W_T\mu e_T(0))\nu_N + W_T\mu \lambda_{N0} \left[ \frac{\partial e_T}{\partial q} \right]_0 \dot{\gamma} = 0$$
$$\gamma_N = w_N^T \dot{\gamma} = 0,$$  

for which the second derivative $\gamma_N$ of the gap function $g_N$ has to be used. For the final representation of the first order dynamics, one eliminates $\nu_N$ from the first equation in (7) and switches then to a new set of minimal coordinates $z$ to remove artificial asymmetries caused by this process. With $y = Qz$ and $z$ such that $g_N = w_N^T y = w_N^T Qz \equiv 0 \forall z$, one obtains the classical orthogonality condition $w_N^T Q = 0$, and reduction with the help of virtual work yields

$$Q^T M^T Q \dot{z} + Q^T H Q \dot{z} + Q^T K Q \dot{z}$$
$$+ Q^T (\frac{\partial}{\partial q} W_T e_T(0) w_N^T M^{-1})(H Q \dot{z} + K Q \dot{z}) = 0.$$  

Although $H$ can be shown to be symmetric and positive definite, there are non-removable asymmetries caused by the term $\frac{\partial}{\partial q} W_T e_T(0) w_N^T M^{-1}$. They affect both, the new damping and stiffness matrix, and may therefore cause the equilibrium to become unstable.

RESULTS

For the four wheels at the leading bogie of the rail car, various configurations have been tested for stability by numerically varying the friction coefficient $\mu$ and the nominal creep direction $e_T(0)$. Figure 1 shows these results for a driving velocity of 4.2 m/s and a wheel diameter of 800 mm. Stable regions are marked in light grey, unstable regions in dark grey. Steady sliding equilibria which correspond to real driving states are shown as black dots. In particular the front inner wheel is prone to instability, which has been confirmed by measurements and additional numerical simulations of the full underlying non-smooth model. The latter revealed a stick-slip limit cycle of about 4000 Hz in the case of an unstable equilibrium point.

REFERENCES