Abstract

Curve squealing of railroad vehicles occurs when trains run through narrow curves and a given set of atmospheric and mechanical conditions are present.

The modal characteristics of the wheel play an important role in this phenomenon and have therefore been studied. The numerical as well as the experimental determination of its eigenforms and eigenvalues have been performed. The modal damping coefficients of the wheel have also been measured.

When curve squealing occurs, it is supposed that a limit cycle between wheel and rail is established. In order to numerically simulate curve squealing, a general multi-body contact problem with Coulomb friction and unilateral contact is set up. The dynamic response of the wheel is evaluated by superposition of its modes. The time-integration is performed with a time-stepping method and the solution of the contact problem is conducted with the Augmented Lagrangian algorithm.

Key words
Squealing, railroad, experiments, Coulomb friction, self-excited oscillations, Contensou friction.

1 Introduction

Curve squealing of railroad vehicles is a well known problem and has been studied frequently in literature ([Remington, 1987; van Ruiten, 1988; Fingberg, 1990; Périard, 1998; Heckl, 2000; Beer, 2000]). When investigating curve squealing, one first has to identify the set of conditions for which it occurs. This is not an easy task because both atmospheric and mechanical aspects have to be taken into account. Long-time field measurements [Stefanelli, 2003] have revealed serious difficulties. For example, similar trains running through the same curve on the same day may have a very different behavior with respect to curve squealing.

The wheel plays a central role in curve squealing and that is why it has been extensively studied ([Thompson, 1993; Ugolotti, 2000]).

In the present work, measurements and Finite Element (FE) simulations have been performed in order to determine the modal characteristics of the wheel. FE simulations and measurements have also indicated how the eigenfrequencies of the wheel depend on the wheel diameter [Cataldi, 2004]. The modal damping coefficients of the wheel have also been experimentally determined. Furthermore, in order to simulate curve squealing, a general multi-body contact problem with Coulomb friction is set up [Glocker, 2001]. The elasticity of the wheel is considered by evaluating its dynamic response as superposition of its modes and the time-integration is performed with a time-stepping method. The solution of the contact problem is conducted with the Augmented Lagrangian algorithm [Leine, 2004].

2 Modal characteristics of the wheel

The typical frequency spectrum of a squealing wheel is characterized by the presence of one predominant frequency. This suggests that only a few eigenforms are involved. The modal characteristics of the wheel have therefore been studied. The wheel has a weight of 258 kg and a nominal diameter of 820 mm (Figure 1).

Figure 1. The studied wheel mounted on a driving trailer of the Swiss Federal Railways.

Due to abrasion, the profile of the wheel deteriorates
and, for safety and comfort reasons, the wheel rim needs regular maintenance reprofiling, which causes a reduction of its thickness. The minimal wheel diameter value that is allowed is 760 mm. The resulting geometrical variation induces both stiffness and mass reductions which can not be neglected.

2.1 FE model
In field measurements [Stefanelli, 2002], squealing frequencies up to about 5kHz have been recorded for this kind of wheel. Therefore the number of the elements and their distribution have been optimized in order to obtain reliable data at least up to 6 kHz [Cataldi, 2003] (Figure 2).

![Figure 2](image_url) Left: a radial cross-section of the used mesh. Right: the full FE model of the wheel. Both pictures refer to the wheel with a diameter of 820 mm.

2.2 Measurement set-up
The experimental modal analysis has been performed using a laser scanning vibrometer. With this device it is possible to define a measurement grid on the wheel surface and automatically measure at all points. As the excitation signal and the measurement signal are synchronised, it is possible to find the phase difference between the two signals. Therefore, all the measured points can be combined together to obtain the modal shapes and frequencies.

The wheel has been excited perpendicularly to its plane using a piezo element fixed at its rim. The excitation signal used was a periodic chirp with frequencies up to 10 kHz. This measurement set-up is summarized in figure 3.

![Figure 3](image_url) Overview of the experimental set-up used for determining the modal shapes and frequencies of the wheels.

2.3 Measurements
In order to test the validity of the FE model, the calculations regarding a free wheel have been validated experimentally (figure 4, left). Subsequently, the presence of the wheelset axle has been considered by fixing the central nodes in the FE Model. In this way more realistic boundary conditions for the wheel are achieved. This case has also been validated by full scale measurements performed on a mounted wheel (figure 4, right).

![Figure 4](image_url) Left: laboratory measurements on a "free" wheel. In foreground the head of the Laser Scanning Vibrometer (PSV-300) used for the measurements. Right: measurements on a "mounted" wheel.

2.4 Eigenfrequencies-shift
Using the validated FE Model, it has been possible to evaluate the shift in the eigenfrequencies. The eigenfrequencies have been calculated for wheel diameters of 820 mm, 800 mm, 780 mm and 760 mm. That diameter reduction, only occurs due to the reduction of the wheel tire thickness (see figure 7).

The reduction of the wheel tire thickness has two effects. One, is the reduction of the wheel tire mass. The
other, is a reduction in the wheel tire stiffness. Both are likely to be involved in modal characteristics changes.

The wheel sections used are presented in figure 8. For

given eigenforms, a variation of their frequencies up to 15% have been found [Cataldi, 2004] (Figure 9). Each line in figure 9 connects the eigenfrequency corresponding to the same eigenmode identified for the various wheel diameters. These results have also been validated by measurements. Figure 10 shows the Fast Fourier Transform (FFT) of measurements made on a new wheel (dotted line) and on a fully worn wheel (heavy line). It is possible to see how the frequency of a given mode may change. As expected, the modes by which the wheel tire oscillates undergo a frequency change when the wheel diameter is varied. On the other hand, the modes with no wheel tire oscillation do not present significant changes in their eigenfrequencies.

2.5 Modal damping measurements

The modal damping coefficients have been measured in order to better reproduce the dynamical response of the wheel. The experimental set-up is described in figure 11. The measurement is controlled by a labview program. The excitation frequency is sent to a function generator and after having being amplified it reaches a piezoelement attached to the wheel. The wheel vibration is then measured by a laser. After having been filtered, the signal enters in the phase meter where the phase difference with respect to the reference signal is measured. The amplitude of both signals are shown with the use of an oscilloscope. The computer collects the frequency, the phase-shift and the amplitude information by a General Purpose Interface Bus (GPIB) connection with the various instruments. Due to the relatively low damping ratio of the wheels, the eigenfrequencies are enough apart allowing to treat them separately. Furthermore, a linear behaviour of the eigen-
modes and a 1-degree of freedom (1-DOF) model, for each eigenmode, has been assumed. The damping ratios have subsequently been extracted from both phase-frequency and amplitude-frequency curves. Figure 12 shows an example in which the 1-DOF model well agrees. In some cases this method could not be applied due to the proximity of more eigenfrequencies. A multi-degrees of freedom approach will therefore be needed for such cases. The measured dimensionless modal damping values have a magnitude of about 0.0002-0.0005 and well agree with the values given by [Thompson, 1993].

3 Numerical Simulation

In this section a mathematical non-linear model which will be used to simulate the curve squealing of a railway wheel is presented. It has been observed that when squealing occurs, it has a rather stable behaviour, i.e. squealing is present all along the curve. In the mathematical model the wheel is considered to be fixed and in contact with a surface (representing the rail) which has a relative velocity with respect to the wheel. The influence of the atmospheric conditions is integrated in the friction coefficient.

The time-integration is conducted with a time-stepping method [Moreau, 1988]. The highest eigenfrequency considered gives an upper limit to the value of the integration step, while the lower eigenfrequency gives a lower limit to the time for which the simulation has to last.
3.1 Parameters of the simulation
In order to model the curve squealing, the mechanical properties of the wheel are necessary but not enough. The magnitude of the forces acting from the train on the wheel, as well as the position of the contact point and the angle of attack are important. For this purpose the steady state solution of the train running through the curve is considered. From this solution it is also possible to get the relative velocities (tangential, longitudinal and spin) at the contact point (figure 13).

3.2 Unilateral contact with Coulomb friction
A general multi-body unilateral contact problem with Coulomb friction can be expressed by the following set of differential equations [Glocker, 1995; Pfeiffer, 1996]

\[ M\ddot{u} - h - W_N\lambda_N - W_T\lambda_T = 0, \]  
(1)

and the set-valued force laws

\[ -\lambda_N \in \text{Upr}(g_N(u, t)), \]  
(2)

\[ -\lambda_T \in \mu\lambda_N \text{Sgn}(\dot{g}_T(u, q, t)), \]  
(3)

for the normal (2) and the tangential (3) direction. The variable \( t \) indicates the time dependence, the generalized coordinates of the multi-body system are gathered in the vector \( q(t) \) and the velocities of the generalized coordinates are gathered in the vector \( \dot{q}(t) \). \( M(q, t) \) is the mass matrix of the system and the vector \( h(q, u, t) \) contains all the smooth forces acting on the system. The matrices \( W_N(q, t) \) and \( W_T(q, t) \) are the generalized contact force directions (in normal and tangential direction) with \( \lambda_N \) and \( \lambda_T \) being their values. The Upr represent the unilateral contact, the Sgn represent a force with given minimal and maximal value, \( g_N \) expresses the separation in the normal direction of two possible contact points and \( \dot{g}_T \) expresses the relative tangential velocity of two possible contact points.

The elastic response of the wheel is simulated using a FE Model. The governing equation of a dynamical FE model is given by:

\[ \ddot{\bar{z}} + \bar{C}\dot{\bar{z}} + \bar{K}\bar{z} = f, \]  
(4)

where the vector \( z \) contains the generalized displacements of the nodes of the FE model, \( \bar{M} \) is the mass matrix, \( \bar{C} \) the damping matrix, \( \bar{K} \) the stiffness matrix and \( f \) the vector of generalized external forces acting on the model. The number of degrees of freedom (dof) of the FE Model is usually too large to directly consider the dof of the FE model as generalized coordinates in the multi-body system. Therefore a suitable way to combine both models is necessary and is presented in the following section.

3.3 Modal Superposition
The dynamic response of a structure can be described by a superposition of the structural modes [Cook, 1981; Sextro, 2002]. The generalized displacements \( z \) of the FE Model are then expressed as:

\[ z = \Phi q, \]  
(5)

where the modal matrix \( \Phi \) is formed by the chosen normalized eigenvectors and the vector \( q \) contains the respective modal amplitudes. The virtual work variation for an arbitrary virtual displacement \( \delta q \) of the FE model is given by

\[ \delta z^T(\bar{M}\ddot{\bar{z}} + \bar{C}\dot{\bar{z}} + \bar{K}\bar{z} - f) = 0. \]  
(6)

We now insert (5) into (6) and consider arbitrary variations \( \delta z \), which gives

\[ \Phi^T\bar{M}\Phi \ddot{\bar{q}} + \Phi^T\bar{C}\Phi \dot{\bar{q}} + \Phi^T\bar{K}\Phi q = \Phi^T f. \]  
(7)

Taking mass normalized eigenvectors yields to the reduced mass and stiffness matrices:

\[ \bar{M} = \Phi^T\bar{M}\Phi = E, \]  
(8)

\[ \bar{K} = \Phi^T\bar{K}\Phi = \text{diag}(\nu_{0j}^2), \]  
(9)

where \( E \) is the unit Matrix and \( \nu_{0j} \) are the eigenfrequencies of the system.

For the damping matrix it is possible to use the Rayleigh assumption and therefore to express it as a combination of the mass and stiffness matrix

\[ \bar{C} = \alpha\bar{M} + \beta\bar{K}, \]  
(10)
which leads to the reduced damping matrix

\[
C = \Phi^T \bar{C} \Phi = \text{diag}(2\omega_0 j C_j),
\]

(11)

where the \(C_j\) are the modal damping coefficients. This leads to the equation of motion

\[
M \ddot{q} + C \dot{q} + K q = \Phi^T f
\]

(12)

which can be put in the form (1). The mass matrix is now an identity matrix, the \(h\) vector contains the terms \((C \dot{q} + K q)\) and the external forces \(\Phi^T f\) are split in the terms \(W_N \lambda_N\) and \(W_T \lambda_T\).

4 Preliminary results

Some simulations have been performed in order to qualitatively test the model. Therefore, the considered values do not yet correspond to operational conditions. The first simulation presents the case in which the wheel undergoes a vertical load, acting on its center, and a sinusoidal load in the longitudinal direction, acting on the contact point. The vertical load has a constant magnitude of 10000 N, the sinusoidal load has an amplitude of 7000 N and the friction coefficient has been set to 0.5, corresponding to a dry rail-wheel contact. In figure 14 it is possible to see how the system behaves. As soon as the longitudinal force reaches the limit value of 5000 N, the contact point starts to move. It then stops as soon as the magnitude of the longitudinal force decreases under the limit value of 5000 N. The dynamical system needs some time to reach a stationary behaviour. This can be seen in figure 15. This simulation indicates how the model correctly shifts from a sticking to a sliding condition.

The second simulation has the same vertical load and friction coefficient as the first but no longitudinal force.

Instead, there is a given relative velocity in the longitudinal direction at the contact point. The value of the relative velocity is 0.001 m/s. In figure 16 it can be seen that the system moves away from the starting configuration and starts to oscillate chaotically but in a limit-cycle-like fashion. The aim of this second simulation is to show how the system can become self-excited due to sliding contact.

We conclude that, if it is possible to find the set of conditions leading to curve squealing, then this simulation program should be able to reproduce the limit-cycle associated with squealing.

5 Conclusions

A large number of measurements has been performed in order to validate the results of the FE calculations.
Also the modal damping values have been experimentally determined with the aim to better qualify the mechanical characteristics of the wheel.

A new possible approach to curve squealing simulation has been presented, together with some preliminary results. It has been shown that the mathematical model is able to reproduce a limit-cycle-like behaviour also considering Coulomb-friction.

6 Further research

The present model can only consider a longitudinal and a lateral force at the contact point. The model will be extended with a pivoting friction torque by considering the Coulomb-Contensou friction [Leine, 2003]. Some eigenmodes of the rail present shapes by which the rail-head vibrates perpendicularly to the railway. At the nodes of such eigenmodes a torsion occurs and it cannot a priori be excluded that this also has an effect on the squealing phenomenon. Therefore, the consideration of a pivoting friction torque will give a more general simulation tool.

With a commercial multi-body program it will be possible to simulate the run of the railroad vehicle through the curve. Parameters such as the contact point position and the relative velocities at the contact point will then be available. They will be used as starting configuration for the investigation of the curve squealing using the presented model.

The eigenforms which are supposed to be involved in the noise emission, are those which cause the wheel rim to vibrate. Therefore the wheel axle has not yet been modeled. Further investigations also considering the wheel axle are not excluded.

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