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Positioning of small particles by an ultrasound field excited by surface waves

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Abstract

A method for the controlled positioning of small particles in one or two dimensions by an ultrasound field excited by a surface wave is presented. Particles of a diameter between 10 and 100 μm placed on a surface can be concentrated at certain locations and moved over the surface. In other approaches it is possible to let the particle levitate freely in the fluid. However for the use of ultrasonic positioning in for example microassembling it is necessary to move particles over a surface as well as to let them levitate over the surface.

Physical principle: A two- or three-dimensional ultrasound field is excited in a fluid filled gap between a rigid surface at the bottom and a vibrating surface of a solid at the top. The height of the gap varies between 0.1 and 2 mm. A one-dimensional sinusoidal vibration of the upper surface excites a two-dimensional ultrasound field in the fluid. Particles that are arbitrarily distributed on the lower surface will be concentrated in lines by the ultrasound field.

First the calculation of the field of forces on particles in the fluid layer is presented. Then the dispersion relation of a vibrating plate which is in contact with a fluid on one side is derived. The technical setup will be introduced. Finally the experiments are shown and compared to the theoretical results.

Keywords: Micromanipulation; Positioning; Ultrasound; Surface wave

1. Introduction

A method of manipulating small particles is presented which uses forces that act on an object in an acoustical field (acoustical force). To apply theses forces a two- or three-dimensional sound field is used. This sound field is excited by a plate which vibrates. Its surfaces perform sinusoidal movements and it emits an acoustical wave into a layer of fluid. This wave is reflected by a rigid surface and generates a standing sound field in the fluid. In this sound field acoustical forces act on particles. The particles can be concentrated and displaced in one, two or three dimensions.

The advantage of this technique lies mainly in three points, (i) particles of a diameter between one and some hundred micrometer can be manipulated, (ii) handled in a contactless manner and (iii) many particles can be manipulated simultaneously by using the repetitive structure of the sound field. This makes it suitable for applications in micro- and biotechnology.

The introduced approach is a consequence of a development which started in the first half of the 20th century. An important idea, which is already used industrially, deals with the separation of particles from a surrounding fluid [1]. In the last decade many publications treated the handling of a few or single particles. Often multiple transducers were used to excite a specially shaped sound field in which particles can be moved arbitrarily [2,3].

2. Theory

The setup is shown in Fig. 1. The sound field in the fluid is emitted by a plate which is excited to mechanical vibrations by a transducer. In this section the force distribution in the fluid layer alone and then the dispersion relation of the plate–fluid combination will be considered.

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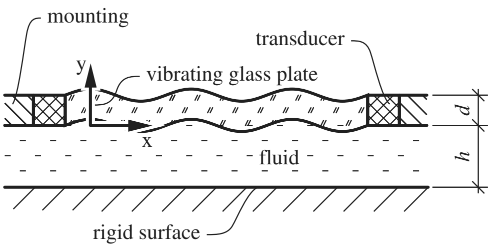

Fig. 1. Design of the apparatus. It consists of a plane glass plate (drawn performing vibrations) with the thickness d and a fluid layer of height h underneath.

2.1. Force field in the fluid layer

First for the calculation of the sound field it shall be assumed that it is excited by a spatially harmonic displacement at $y = 0$ and reflected by a rigid plane at $y = -h$ (Fig. 1). For the two-dimensional sound field in the fluid the displacement of the surface in y-direction can be written as, $u_{\mathrm{Sf}} = u_{\mathrm{Sf0}} \cos(xk_{\mathrm{Sf}})\mathrm{e}^{\mathrm{i}\omega t}$, where u_{Sf0} is the displacement amplitude, k_{Sf} the wave number of the surface displacement and ω the angular frequency. The sound field in the fluid is a superposition of four plane acoustical waves $\phi_{\mathrm{F}} = \sum_{m=1}^{4} \phi_m$. Each of them can be described by its velocity potential $\phi_m(x, y) = \Phi_m \mathrm{e}^{\mathrm{i}(\omega t \pm k_{\mathrm{F}_x} x \pm k_{\mathrm{F}_y} y)}$, where k_{F_x} and k_{F_y} are the wave numbers in the fluid in the x- and y-direction, respectively, and are positive. In the case of a lossless fluid, which is regarded here, the amplitudes of these four waves are equal $\Phi_m = \Phi$. Out of the four waves always two propagate in opposite direction and form a standing wave. The sound field results from two boundary conditions. At $y = 0$ the fluid movement in y-direction is given by the surface movement u_{Sf} and at $y = -h$ the fluid movement must vanish

$$-\left.\frac{\partial \phi}{\partial y}\right|_{y=0} = \left.\frac{\partial u_{\mathrm{Sf}}}{\partial t}\right|_{y=0} \quad \text{and} \quad \left.\frac{\partial \phi}{\partial y}\right|_{y=-h} = 0.$$

The resultant sound field in the fluid layer is than given by

$$\Phi_{\mathrm{F}} = \frac{\omega u_{\mathrm{Sf0}}}{k_{\mathrm{F}_y}} \frac{\mathrm{e}^{2hik_{\mathrm{F}_y}}}{1 - \mathrm{e}^{2ihk_{\mathrm{F}_y}}} (\mathrm{e}^{\mathrm{i}yk_{\mathrm{F}_y}} + \mathrm{e}^{-\mathrm{i}k_{\mathrm{F}_y}(y+2h)}) \cos(xk_{\mathrm{F}_x})\mathrm{e}^{\mathrm{i}\omega t}, \tag{1}$$

where k_{F_x} is the wave number of the sound field in x-direction; it coincides with the wave number of the surface $k_{\mathrm{F}_x} = k_{\mathrm{Sf}}$. k_{F_y} is the wave number in y-direction, perpendicular to the surface. It is given by $k_{\mathrm{F}_y} = (k_{\mathrm{F}}^2 - k_{\mathrm{Sf}}^2)^{1/2}$, where k_{F} is the wave number in the fluid. When the wave length in the fluid is longer than in the surface, that means $k_{\mathrm{F}} < k_{\mathrm{Sf}}$, there is no sound emission; k_{F_y} becomes purely imaginary leading to an exponential decay of the fluid movement away from the surface. This is called "emission condition". It states that for sound emission the wave length in the fluid must not be smaller than the one of the surface disturbance.

In [4] a formula for the mean force $\langle \vec{F} \rangle$ on a sphere in an arbitrary sound field is presented. It results from a force potential U with $\langle \vec{F} \rangle = -\nabla U$. This force potential is

$$U = 2\pi r_{\mathrm{S}}^3 \rho_{\mathrm{F}} \left(\frac{1}{3} \frac{\langle p^2 \rangle}{\rho_{\mathrm{F}}^2 c_{\mathrm{F}}^2} f_1 - \frac{\langle q^2 \rangle}{2} f_2 \right) \tag{2}$$

with $f_1 = 1 - \rho_{\mathrm{F}} c_{\mathrm{F}}^2 / (\rho_{\mathrm{S}} c_{\mathrm{S}}^2)$ and $f_2 = 2(\rho_{\mathrm{S}} - \rho_{\mathrm{F}})/(2\rho_{\mathrm{S}} + \rho_{\mathrm{F}})$, where ρ_{F} and ρ_{S} are the densities of the fluid and of particle, respectively, c_{F} and c_{S} are the longitudinal wave speeds in the two materials and r_{S} is the particle radius. $\langle p^2 \rangle$ and $\langle q^2 \rangle$ are the mean square fluctuations of the pressure and velocity in the fluid at the location where the particle is assumed to be. They are calculated as the time averaged squares of the real parts of pressure and magnitude of velocity. The locations where the particles are finally concentrated are given by the minimum of the force potential. The positions of these minima and maxima of U depend on the spatial coordinates. The sound field shall be three-dimensional and is described by

$$\phi_{\mathrm{F}} = \frac{\omega u_{\mathrm{Sf0}}}{k_{\mathrm{F}_y}} \frac{\mathrm{e}^{2hik_{\mathrm{F}_y}}}{1 - \mathrm{e}^{2ihk_{\mathrm{F}_y}}} (\mathrm{e}^{\mathrm{i}yk_{\mathrm{F}_y}} + \mathrm{e}^{-\mathrm{i}k_{\mathrm{F}_y}(y+2h)}) \cos(xk_{\mathrm{F}_x}) \times \cos(zk_{\mathrm{F}_z})\mathrm{e}^{\mathrm{i}\omega t}. \tag{3}$$

The extrema of the force potential of the velocity potential of Eq. (3) lie on a three-dimensional grid at certain combinations of $xk_{\mathrm{F}_x} = zk_{\mathrm{F}_z} = m\pi/2$ and $(y + h)k_{\mathrm{F}_y} = n\pi/2$ $(m, n \in \mathbb{Z})$. The condition that an extremum is a minimum is that the Hesse matrix of U is positive definite; and this depends on the material combination of fluid and particle. For a material combination of $\rho_{\mathrm{S}} c_{\mathrm{S}}^2 / \rho_{\mathrm{F}} c_{\mathrm{F}}^2 > 1$ and $\rho_{\mathrm{S}}/\rho_{\mathrm{F}} > 1$, which is for example the case of a glass sphere in water, the two-dimensional force field is given in Fig. 2. The height h of the fluid layer is $h = 0.75\lambda_y$, where $\lambda_y = 2\pi/k_{\mathrm{F}_y}$ is the

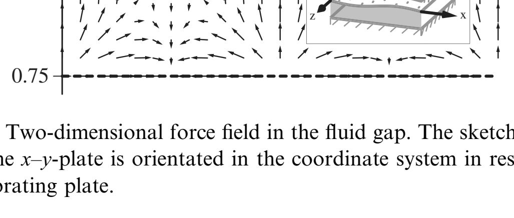

Fig. 2. Two-dimensional force field in the fluid gap. The sketch shows how the x–y-plate is orientated in the coordinate system in respect of the vibrating plate.

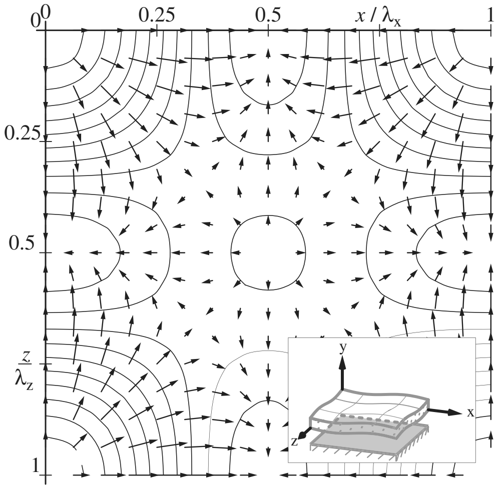

Fig. 3. Force distribution of the on the reflecting surface at $y = -h$ for a three-dimensional sound field. The sketch shows how the x–z-plate is orientated in the coordinate system in respect of the vibrating plate.

wave length in the fluid in the y-direction (in x-direction analogous). It can be seen, that particles are concentrated at $x/\lambda_x = 1/4 + m/2$ and $y/\lambda_y = 1/4 + n/2$ $(m, n \in \mathbb{Z})$.

For the micromanipulation it is of special interest to manipulate particles that are in contact with the surface at $y = -h$. All particles that are heavier than the fluid will sink to this surface by their weight. There they can be collected at certain positions, in Fig. 2 for instance at $x/\lambda_x = 1/4 + m/2$. It should be kept in mind that particles can only levitate in the fluid when the sound field is strong enough. Fig. 3 shows the force distribution on the reflecting surface at $y = -h$ for a three-dimensional sound field. The arrows represent the force vectors and the curves are isolines of the potential of forces U. It can be seen, that the particles are forced to points where $x/\lambda_x = 1/4 + m/2$ and $z/\lambda_z = 1/4 + m/2$. The isolines show, that the minima are not conus-shaped but ellipsoidal. Single particles will be moved to the minimum points of the potential of forces, but a plurality of particles will be collected in longish regions enclosed by a isoline.

2.2. Plate fluid combination

The sound field in the fluid is excited by a vibrating plate, whereby one of its surfaces is in contact with the fluid. As it could be seen in the previous section, the particles are concentrated according to the loops and nodes of the surface movement of the plate as there is an interaction between fluid and plate. In this section the dispersion relation of the wave speed in the plate–fluid combination $c_P(f)$ will deduced. The geometry and coordinate system is shown in Fig. 1. The plate has a thickness of d, the fluid gap has a height of h and their interface is at $y = 0$. It is assumed that the plate and the fluid gap spread infinitely in x-direction and that all materials are lossless.

Fluid loading of a plate was considered by [5] but there the fluid has a free surface. In the following section the two-dimensional case is investigated where the fluid has the vibrating plate on one side and a rigid surface on the other. Fluid and plate move in the x–y-plane. The sound field can be described by the velocity potential of Eq. (1). But to be consistent with the common formulation of Lamb waves it will be written as a displacement potential of a propagating wave

$$\theta_F = \frac{\Theta_F}{e^{-2ihk_{F_y}} - 1}\left(e^{iyk_{F_y}} + e^{-i(y+2h)k_{F_y}}\right)e^{i(\omega t - xk_P)}, \tag{4}$$

where Θ_F is the amplitude and k_P the wave number in the plate (in the former section called k_{Sf}). In the plate the displacement potentials for the P wave θ_P and the S wave are ψ_P are

$$\left.\begin{array}{l} \theta_P = [\Theta_s \cosh qy + \Theta_a \sinh qy]e^{i(\omega t - xk_P)} \\ \psi_P = [\Psi_a \cosh sy + \Psi_s \sinh sy]e^{i(\omega t - xk_P)} \end{array}\right\}, \tag{5}$$

where $q = (k_P^2 - k_l^2)^{1/2}$ and $s = (k_P^2 - k_t^2)^{1/2}$. k_l and k_t are the wave numbers corresponding to the P and S waves, respectively. The displacement in the plate in x-direction is $u_Px = \partial\theta_P/\partial x - \partial\psi_P/\partial y$ and in y-direction is $u_Py = \partial\theta_P/\partial y + \partial\psi_P/\partial x$. From this displacement the stresses in the plate σ_{yy} and σ_{xy} can be calculated in function of the displacement potentials of Eq. (5).

The boundary and transition conditions of this system are: (i) no normal stress at the upper surface of the plate, (ii) and (iii) no shear stress at both surfaces of the plate, (iv) the normal stress of the plate equals the negative liquid pressure at the interface and (v) equal displacement of plate and liquid at the interface. The boundary condition for the rigid surface bordering the fluid at $y = -h$ is already contained in the displacement potential θ_F. Therefore it does not has to be taken into consideration again.

To receive the dispersion relation, σ_{yy} and σ_{xy} are inserted in the five boundary and transition conditions. This set of equation has a nontrivial solution when the determinant is zero; the characteristic equation is

$$\begin{aligned} 0 = \Big[& 16k_P^4 q^2 s^2 + (k_P^2 + s^2)^4 \\ & - 4qs\left(\tanh\left(\frac{d}{2}s\right)\coth\left(\frac{d}{2}q\right) \right. \\ & \left. + \tanh\left(\frac{d}{2}q\right)\coth\left(\frac{d}{2}s\right)\right)k_P^2(k_P^2 + s^2)^2\Big] \\ & + \left\{\frac{\rho_F}{\rho_P}\frac{q}{k_{F_y}}\left((k_P^2 + s^2)^2\coth(dq)\right.\right. \\ & \left.\left. - 4qsk_P^2\coth(ds)\right)k_t^2(k_P^2 - s^2)\cosh(hk_{F_y})\right\}, \end{aligned} \tag{6}$$

where ρ_P and ρ_F are the densities of the plate and fluid. Further calculation would show, that the first term of Eq. (6) enclosed in square brackets is the product of the known characteristic equations of the symmetrical and antisymmetrical Lamb waves. If there is no fluid (i.e. $\rho_F = 0$) Eq. (6) gives the dispersion curves for Lamb waves. With the fluid layer the symmetrical and antisymmetrical modes are coupled and are not independent of each other, because of the second term of Eq. (6) with the density ratio ρ_F/ρ_P as leading factor (enclosed in curly brackets). However there is also decoupling of the symmetrical and antisymmetrical modes when $\cosh(hk_{F_y}) = 0$ resp. $hk_{F_y} = (1/2 + m)\pi$. This means that the fluid layer itself is in resonance. Three cases for plate–fluid coupling can be differentiated and will be considered below: (i) the plate is dominating, (ii) neither the plate nor the fluid dominate and (iii) the fluid dominates.

(i) One reason, why the plate dominates could be that the density ratio ρ_F/ρ_P is close to zero. Because this is trivial and not relevant it is disregarded. More important is the case where the fluid layer is very thin, i.e. $hk_{F_y} \ll 1$ resp. $\cosh(hk_{F_y})$ is close to 1. The thickness of the fluid layer has a certain influence which hardly depends on the frequency. Fig. 4 shows the dispersion curves for a 1 mm glass plate and a 0.1 mm water layer. The gray lines represent the case without fluid (symmetrical and antisymmetrical Lamb waves) and the black curves are with fluid loading. Two facts are remarkable. First the fluid loading has a small influence as expected. Second there seems to be no dispersion curve below the speed of sound in the fluid c_F (in the example $c_F = 1481$ m/s). This points might be related to the "emission condition", which was mentioned above. When the plate waves are slower than the sound in the fluid their wave length would be smaller and therefore no sound emission would be possible.

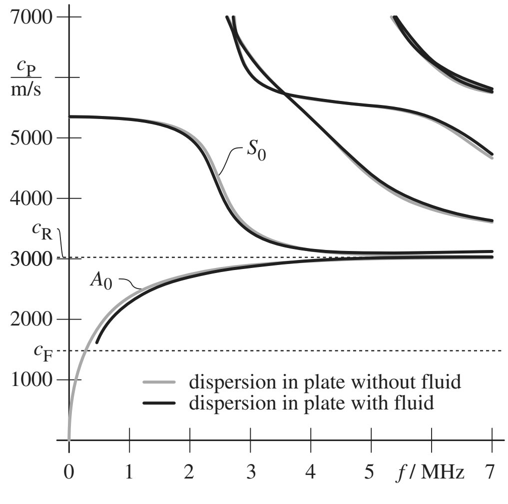

Fig. 4. Dispersion curves of a plate–fluid combination, where the plate dominates the vibration. The plate (glass) is 1 mm thick and the fluid gap (water) has a height of 0.1 mm.

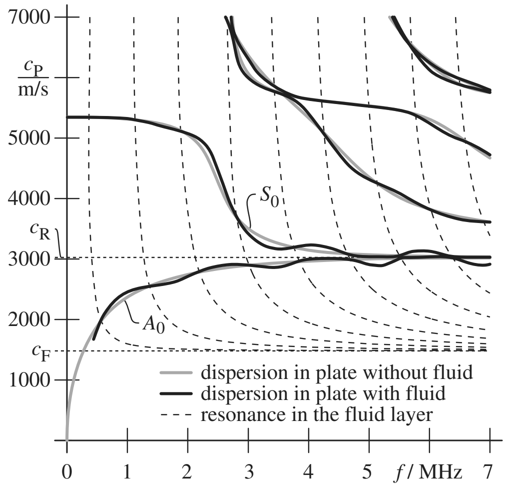

Fig. 5. Dispersion curves of a plate–fluid combination, where neither the plate nor the fluid dominates the vibration. The plate (glass) is 1 mm thick and the fluid gap (water) has height of 1 mm. The dashed line represent resonance in the fluid layer.

(ii) A strong interaction between fluid and plate causes an intense coupling of the different modes in the plate. In Fig. 5 the dispersion relation of a 1 mm glass plate and a 1 mm water layer is displayed. The black curves show the case of fluid loading. It can be seen, that the coupling causes a deviation of the vibration modes from the unloaded case (gray curves). But also here a decoupling can occur, namely when the fluid layer is in resonance. This case is drawn with dashed line. When the resonance condition for fluid layer is fulfilled the dispersion curves of the plate with and without fluid loading intersect.

(iii) When the plate is so thin that its mass can nearly be neglected, the fluid will dominate the "dispersion curves". Fig. 6 shows the case for 1 μm silicon nitride plate with fluid loading of a 0.1 mm water layer. The zeroth antisymmetrical lamb mode without fluid loading disappears with fluid loading. The reason is clear; its wave speed lies under that of the fluid. However a new mode appears, which is called F_1. This is not a real mode. Its nature becomes clear when its vicinity to the mode of pure fluid resonance (dashed line) is regarded. The dashed curves represent the case when the fluid layer without plate is in resonance. This means that the plate swims like an oil film on the fluid and follows its displacements. In this case the plate hardly will excite a sound field in the fluid, but the fluid will force its movements on the plate.

The consequences for the excitation of a sound field for the materials considered can be summarized in three

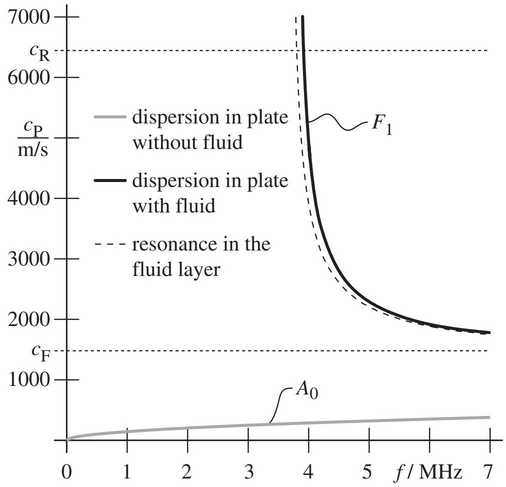

Fig. 6. Dispersion curves of a plate–fluid combination, where the fluid dominates the vibration. The plate (silicon nitride) is 1 μm thick and the fluid gap (water) has height of 0.1 mm.

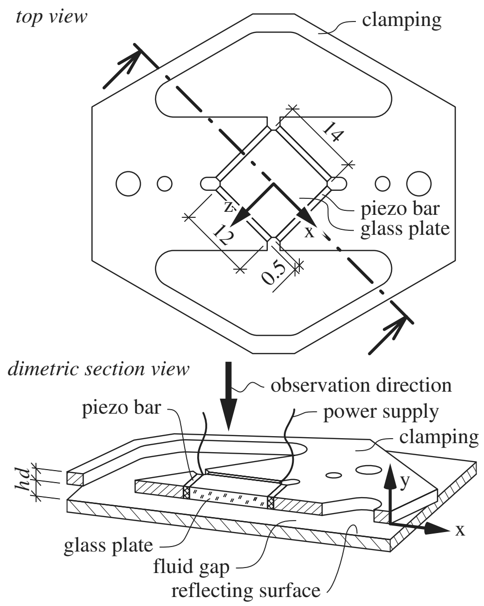

Fig. 7. Setup of the apparatus.

points: First, there seems to be no wave speed in the plate under the speed of sound in the fluid. Second, when the plate is much thinner than the height of the fluid gap, it has to be checked carefully whether it is possible to excite a sound field in the fluid by the plate. Third, when the thickness of the fluid gap is thinner or approximately as thick as the plate, it is sufficient to calculate the dispersion relation only for the plate. The deviation of the dispersion curves with fluid loading is not significant.

3. Experimental setup

As it was explained above, the sound field is excited by a vibrating plate. The plate is excited by a piezo bar. To be able to look through the plate it is made of glass. Fig. 7 shows the setup used for the experiments. The glass plate is 14 mm by 14 mm by 1 mm. Because zeroth asymmetrical vibration mode shall be excited shear-piezo bars are used. The piezo bars are polarized in the y-direction; the electrodes are on the surfaces facing the glass plate and the clamping.

The glass plate is excited to vibrations by applying an RF-voltage across the piezo bars. In the presented setup the frequency lies between 1 and 2 MHz. For a 1 mm thick glass plate the wave length is then approximately 1.7 mm. When a one-dimensional plate vibration shall be excited only one piezo bar is run; for a two-dimensional vibration two waves are excited, propagating perpendicular to each other.

The full unit, consisting of a the glass plate, piezo bars and power supply, is held by a clamping. This clamping is then placed parallel to an arbitrary rigid surface. This surface is not a part of the apparatus; for instance it can be a silicon wafer, on which objects shall be manipulated. Particles in a fluid between the glass plate and the surface are concentrated in lines or points when the power supply is turned on.

4. Results

The particles used for experimental verification are glass spheres of a diameter between 5 and 60 μm. The height of the fluid gap h is approximately 0.2 mm. The fluid is water. At the beginning the particles are arbitrarily distributed on a surface as shown in the left picture of Fig. 8. After the power supply is turned on the particles align in a plurality of straight lines. These lines are parallel to the piezo bar that excites the plate. This state can be seen in the right part of Fig. 8. However the lines are not accurately equidistant. The reason for this is probably, that the sound field is not fully harmonic.

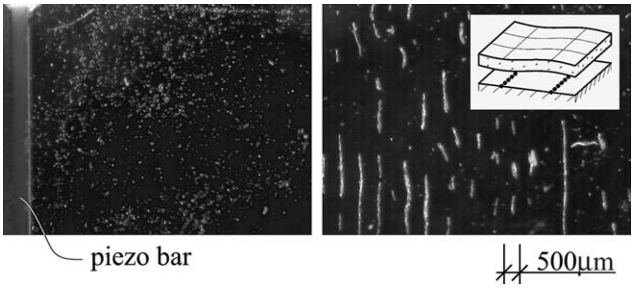

Fig. 8. One-dimensional concentration of glass spheres in a sound field. The left picture shows the particles distributed without sound field, and the right one with sound field. The sketch illustrates the arrangement of the vibrating glass plate and the surface on which the particles are concentrated.

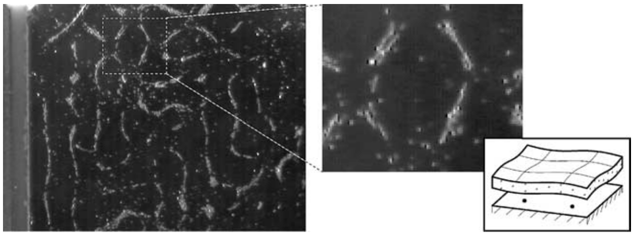

Fig. 9. Two-dimensional concentration of glass spheres in a sound field. The sketch illustrates the arrangement of the vibrating glass plate and the surface on which the particles are concentrated.

The exact shape of the sound field does not only depend on the excitation by the glass plate but also on the shape the fluid. The fluid gap is at the bottom and on the top bordered by the reflecting surface and the glass plate. During the experiment the fluid filled the full space below the glass plate, but lateral boundaries could not be determined exactly. Furthermore the fluid formed a meniscus between the two solid boundary surfaces. Therefore the shape of the fluid is nonuniform and causes that the lines where the particle concentrate are uneven. Fig. 9 show the arrangement of the particles when the sound field is excited with a two-dimensional plate vibration. On the left edge of the picture the particles still form a line like in the one-dimensional case. In this region only a left-rightward running wave exists, because the vertical wave is too weak. Farther away from the edges the particle are concentrated in a grid, whose spacing is equal to the one-dimensional case. The shape of the points where the particles collect is not round, but oval. The oval shape becomes clear when the isolines of the potential of forces on the reflecting plate is regarded. It can be seen in Fig. 3 that around the minima of the potential of forces the isolines are oval. When many particles are concentrated in such a minimum, they assume the form of these oval shaped isoline.

5. Conclusions

In this paper a method to position particles by ultrasound in one or two dimensions is presented. The sound field is emitted by a vibrating plate. This vibration is coupled to a fluid which is on one side in contact with this plate and on the other side with a rigid surface. In the fluid a two- or three-dimensional sound field arises. Particles suspended in the fluid will be concentrated at points where the potential of forces has a minimum. The technical setup comprises a glass plate, shear piezo transducers at its edges and a clamping. This setup can be placed on an arbitrary surface, to position particles on it. The experiments have shown that the particles are arranged in lines or in a two-dimensional grid, as the theoretical investigation predict.

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