Numerical simulations for the time-averaged acoustic forces acting on rigid circular cylinders in ideal and viscous fluids

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Introduction

The time-averaged acoustic force can be applied to many practical fields such as acoustic sensors, ultrasonic levitation and contactless particle manipulation [1, 2] which has become a hot research topic in the field of ultrasonic devices. It is necessary to accurately predict the mean forces on suspended obstacles including the primary force and the secondary force to design ultrasonic particle manipulators. Although there have been many analytical solutions on this topic, it is difficult to determine the acoustic forces on obstacles under more complex system conditions such as proximity to the chamber wall, complex viscous function, acoustic streaming and complicated particle shapes. Therefore, the numerical modelling may become a powerful tool. In this paper, efforts are made to calculate the time-averaged forces on one or two rigid cylinders in ideal and viscous fluids exerted by a standing sound wave field by solving the N-S equations directly using FVM [3] (Finite Volume Method) techniques. The reason why we use FVM here rather than commercial software packages such as COMSOL is that we find FVM method takes much less computation time than them from our preliminary numerical tests. The results are compared with the theoretical prediction, COMSOL simulations and Haydock's [4] simulations. The viscous effects of the host medium are also investigated in detail.

Results in inviscid fluid

We first calculate the cases without viscosity with geometry configurations according to Fig. 1. To make comparison, we also calculate all the cases by a commercial FEM software package named COMSOL. Details of the results are given in Tab.1 for different particle radius.
The results of several methods for the cases without perfectly matched layers (PMLs) are listed in Tab. 1. It can be seen in Tab. 1 that the results between our FVM program and COMSOL agree with each other very well. The maximum relative difference is 3.1%. Meanwhile, the lattice Boltzmann (LB) method [4] gives quite different results compared with FVM and COMSOL. The density contours calculated by our FVM program at time $100T$ are plotted in Fig. 1.

Results in viscous fluid

In Tab. 2, we present the results from FVM with PMLs at the cylinder position $h = 375$ for radius from 5 to 80 with four different viscosities 0.0, 0.167, 0.835 and 1.67. The PMLs eliminate all reflections from the bottom ($y = 0$) and top ($y = y_{max}$) boundaries. The force ratios between the mean forces in the fluid with different viscosities and in the inviscid fluid for different radiuses are plotted in Fig. 2. Obviously, from Fig. 2, the viscosity of the host medium influences the mean forces more significantly at the small cylinders for a certain wave frequency, which is also described by analytical works.
Figure 3 presents the velocity vectors of the acoustic streaming around the cylinder in the case $R = 20$. The streaming field is asymmetric about the cylinder in the horizontal direction, because the cylinder

locates at the midway between a velocity node and its next anti-node. There are four vortices away from the cylinder boundary as described by Nyborg [5].

Tab. 1 Acoustic radiation forces for the cases without PML. (F_{th}: theoretical solutions, F_{LB}: calculated by LB method F_{COM}: calculated by COMSOL, F_{FVM}: calculated by our FVM program)

R	L_y	L_x	h	F_{th} (10^{-5})	F_{LB} (10^{-5})	F_{COM} (10^{-5})	F_{FVM} (10^{-5})
5	100	1000	375	-1.24	-3.04	-1.22	-1.25
10	100	1000	375	-4.95	-8.20	-5.13	-5.17
20	200	1000	375	-19.79	-25.12	-21.13	-21.17
40	200	1000	375	-77.85	-81.02	-108.41	-105.15
80	500	1000	375	-265.70	-247.20	-384.22	-375.30

Tab.2 Mean forces for the viscosity versus the radius ($\times 10^{-5}$)

ν \ R	5	10	20	40	80
0.0	-1.22	-5.09	-19.83	-80.33	-303.96
0.167	-3.01	-8.21	-26.04	-90.48	-316.15
0.835	-4.96	-11.60	-31.89	-102.45	-314.03
1.670	-6.33	-13.89	-33.08	-107.59	-319.17

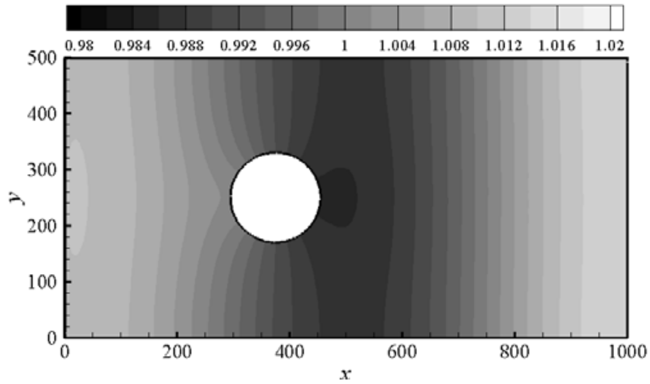

Fig. 1 The density distribution at time 100T.

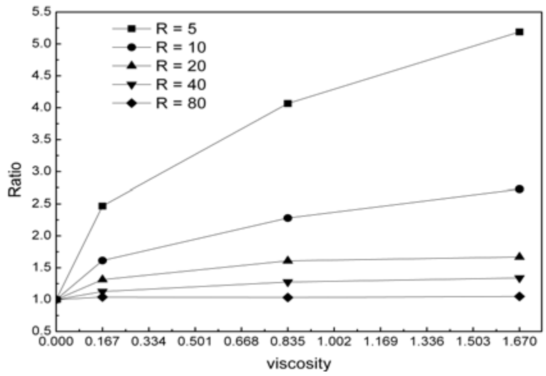

Fig. 2 Force ratios for different cases.

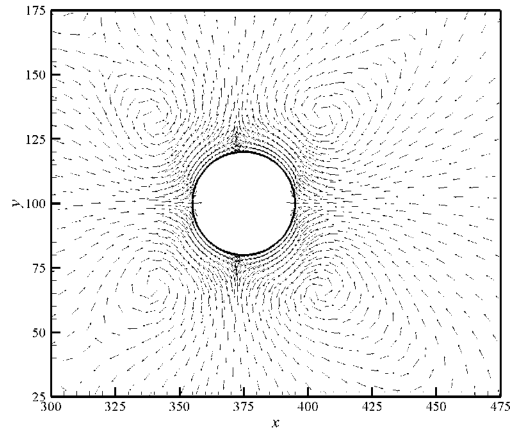

Fig. 3 Velocity vectors of the acoustic streaming around the cylinder (R = 20).

1. Haake, A., et al., *Positioning, displacement, and localization of cells using ultrasonic forces.* Biotechnology and bioengineering, 2005. **92**(1): p. 8-14.
2. Laurell, T., F. Petersson, and A. Nilsson, *Chip integrated strategies for acoustic separation and manipulation of cells and particles.* Chemical Society Reviews, 2007. **36**(3): p. 492-506.
3. Jameson, A. and D. Mavriplis, *Finite volume solution of the two-dimensional Euler equations on a regular triangular mesh.* AIAA paper 85-0435, 1985.
4. Haydock, D., *Lattice Boltzmann simulations of the time-averaged forces on a cylinder in a sound field.* Journal of Physics A: Mathematical and General, 2005. **38**: p. 3265-3277.
5. Nyborg, W.L.M., *Acoustic streaming.* Physical Acoustic, edited by W. Mason (Academic, New York, 1965), 1965. **Vol. IIB**: p. 265-331.