TIME-OPTIMAL TRAJECTORIES OF A DIFFERENTIAL DRIVE ROBOT

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Abstract
The determination of optimal trajectories of non-holonomic systems and of structure-variant (hybrid) mechanical systems are active research areas. These optimization problems belong to the class of mathematical problems with equilibrium constraints (MPEC). In this report the time-optimal trajectories of a structure-variant differential-drive robot will be presented by making use of recent developments in non-smooth analysis and mechanics. The trajectories will be determined by the direct shooting method, which performs the integrations of the dynamical system based on the time-stepping scheme.

Key words
non-smooth, hybrid , non-holonomic, optimal control, MPEC

1 Introduction

The trajectory determination of structure-variant systems which are exposed to high degree of nonlinearity, discontinuities in states and change of degrees of freedom (DOF) is a very challenging active research area. The trajectory optimization of structure-variant mechanical systems belong to the class of mathematical programs with equilibrium constraints (MPEC). In (Outrata and Zowe, 1998) a MPEC is defined as an optimization problem in which the essential constraints are defined by parametric variational inequality or complementarity systems. One of the many representations of a MPEC can be stated as follows:

$$\begin{aligned} \min_{\mathbf{x},\mathbf{z}} \quad & f(\mathbf{x},\mathbf{z}) \\ & \mathbf{z} \in \mathcal{S}(\mathbf{x}) \\ & \mathbf{x} \in \mathcal{U}_{ad},\, \mathbf{z} \in \mathcal{Z} \end{aligned} \tag{1}$$

The problem stated in (1) includes a subclass of so-called bilevel programs, where $\mathcal{S}$ assigns each $\mathbf{x} \in \mathcal{U}_{ad}$ the solution of a "lower-level" optimization problem. In the case where the complementarity system arises from mechanical systems, a so-called subclass of MPEC, namely, bilevel programs apply. In references (Cottle and Stone, 1992), (Murty, 1988) a detailed treatment of complementarities and optimization can be found. By analogy, the measure-differential inclusion that describes the dynamics can be considered as the necessary conditions of a "lower-level" optimization problem represented by the saddle-region restraining set $\mathcal{S}$. Here the time is the goal function to be minimized and the control action is represented by $\mathbf{x} \in \mathcal{U}_{ad}$. References (Luo and Ralph, 1996), (Outrata and Zowe, 1998) treat MPEC and bilevel programs extensively.

The mechanical system chosen to present the numerical scheme has several features. Non-holonomic mechanical systems such as wheeled robotic devices are used regularly in literature as benchmark systems for trajectory optimization such as in (Balkcom and Mason, 2002), (Kolmanovsky, 1995). In all the publications the wheeled robots are treated as smooth mechanical systems that always fulfill the non-holonomic and rolling constraints. These models are either kinematic or seldomly dynamic and are far from being realistic. The differential-drive robot presented in this report will have the ability to undergo stick-slip transitions. Therefore a dynamic model of a three-wheeled robot has been used, tailored for trajectory optimization problems meaning that the level of complexity has been kept adequate. This non-smooth dynamic model is capable of fully incorporating the effects of structure-variance emanating from the non-holonomic constraints. Structure-variance for non-holonomic mechanical systems bear advantages as presented in some typical maneuvers. The controls can be considered as the variables of the "higher-level" optimization problem whereas the contact forces and states are variables of the "lower-level" (quasi-) optimization problem.

The optimization of the MPEC will be performed by the direct shooting method. The discretized control vector and the final time constitute the variable vector. The numerical integrations are performed by applying Moreau's time-stepping discretization scheme to the measure-differential inclusion describing the mechanical dynamics. The contact dynamics arising from a spatial Coulomb friction are handled by a so-called exact-regularization technique that has its roots in the augmented Lagrangian formulation of the Lagrangian dynamics as first described in (Alart and Curnier, 1991). The measure-differential inclusion description is capable of incorporating mechanical impacts as well as stick-slip transitions. In the sequel the measure-differential inclusion representation of the mechanical system will be illustrated by making use of the diff-drive robot from the perspective of trajectory optimization. Moreau's time-stepping discretization scheme along with the exact-regularization will be expounded. The implementation of the optimization with the Nelder-Mead simplex method and the direct shooting technique will be elucidated before presenting results.

2 Model of the Differential-Drive Robot

The differential-drive robot is a three-wheeled motorized robot of which the rear wheels are actuated and steered separately contrary to the front wheel which is neither actuated nor steered. There are several assumptions that are fundamental to the modelling of the system:

1. A rigid-body mechanical model is used.
2. The friction between wheels and ground is modelled as isotropic spatial Coulomb friction.
3. The non-steered unactuated front wheel is equivalently replaced by a stick, removing two DOF to be modelled.
4. The rotational inertia of the total actuation consisting of the components of motor rotors and transmissions are added to the rotational inertias of the wheels.
5. The separation of wheel contact from the floor is excluded but detected. It is assumed that all wheels remain in contact with the floor. The normal contact forces/impulses have been calculated by the the projection of the angular and linear momentums in the constrained directions.

The system has three modii of operation. When the non-holonomic constraints are fulfilled, the system possesses two DOF. If both wheels slide it is a system with five DOF. In the three-DOF mode one wheel contact sticks and the other wheel slides, meaning that the non-holonomic constraints are fulfilled but one wheel does not fulfill the rolling condition. Under the given assumptions there are five mechanical DOF necessary in order to model the mechanical system. These are the planar translational coordinates of center of mass (CM) of the chassis x and y as well as the planar orientation of the chassis ϕ, the angular positions ψ_R and ψ_L of the wheels with respect to the chassis frame. The various dimensional and inertial parameters relevant to model have been obtained from a CAD model of a similar diff-drive robot of this type developed at ETH Zurich. As a consequence, the following set of generalized coordinates and velocities are used to describe the system:

$$\mathbf{q} = \begin{pmatrix} x \\ y \\ \phi \\ \psi_L \\ \psi_R \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \\ \dot{\psi}_L \\ \dot{\psi}_R \end{pmatrix}. \tag{2}$$

Here $\mathbf{u}$ is a function of bounded variation and is called a generalized velocity. The equations of motion (EOM) are obtained by using the well-known Lagrange II formalism for the smooth dynamics of the robot:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\mathbf{q}}}\right)^T - \left(\frac{\partial T}{\partial \mathbf{q}}\right)^T + \left(\frac{\partial V}{\partial \mathbf{q}}\right)^T - \mathbf{f} = \mathbf{0}. \tag{3}$$

The contact forces, which are non-potential in the classical sense, are incorporated by the appropriate jacobians $\mathbf{W}_T$ and $\mathbf{W}_N$ in $\mathbf{f}$. In the sequel mechanical systems with unilateral contacts with friction will be considered but this formalism is capable of modelling a wider variety of mechanical systems as described in (Glocker, 2001) from the perspective of non-smooth potential theory.

The tangential and normal local kinematics need to be defined in order to relate the contact distance to the set-valued force element. For the detection of the closing of a contact, let the vector $\mathbf{g}_N(\mathbf{q}, t)$ denote the normal contact distances between the rigid bodies in the system which are always positive. The tangential and normal contact kinematics are defined in terms of:

$$\boldsymbol{\gamma}_T = \mathbf{W}_T^T(\mathbf{q}, t)\mathbf{u} + \boldsymbol{\omega}_T, \tag{4}$$

where $\boldsymbol{\gamma}_T$ is a vector that denotes the relative tangential contact velocities, and

$$\boldsymbol{\gamma}_N = \mathbf{W}_N^T(\mathbf{q}, t)\mathbf{u} + \boldsymbol{\omega}_N = \frac{\partial \mathbf{g}_N}{\partial \mathbf{q}}\mathbf{u} + \boldsymbol{\omega}_N, \tag{5}$$

where $\boldsymbol{\gamma}_N$ is a vector that denotes the relative normal contact velocities and is obtained as the total time derivative of $\mathbf{g}_N(\mathbf{q}, t)$. In order to derive the set-valued force laws properly the definition of following sets are necessary:

$$C_1 = \{\lambda_N \mid \lambda_N \geq 0\} \tag{6}$$
$$C_2 = \{\lambda_T \mid \quad |\lambda_T| \leq \mu\, \lambda_N\} \tag{7}$$

where λ_T and λ_N represent the associated Coulomb friction force and the normal contact force at one contact, respectively. The differential inclusion can be

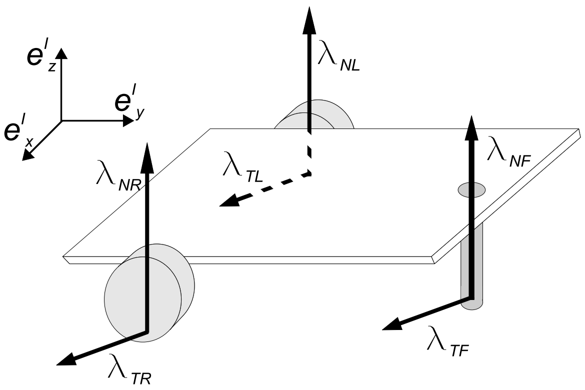

Figure 1. Contact forces at the model contacts.

stated in the following form:

$$\mathbf{M}\,\dot{\mathbf{u}} - \mathbf{h}(\mathbf{q},\,\mathbf{u},\,t) - \mathbf{W}_T\boldsymbol{\lambda}_T - \mathbf{W}_N\boldsymbol{\lambda}_N = \mathbf{0}, \quad (8)$$
$$g_{N_i} = 0, \quad (9)$$
$$\gamma_{N_i} \in -\mathcal{N}_{C_1}(\lambda_{N_i}), \quad (10)$$
$$\gamma_{T_i} \in -\mathcal{N}_{C_2}(\lambda_{T_i}), \quad (11)$$

where $\mathbf{M}(\mathbf{q}, t)$ is the symmetric PD mass matrix and $\mathbf{h}(\mathbf{q},\,\mathbf{u},\,t)$ represents the vector with gyroscopical accelerations together with all classical finite-valued generalized forces. $\mathcal{N}_C(\boldsymbol{\lambda})$ denotes the normal cone to set C at the point $\boldsymbol{\lambda}$. A detailed treatment of the set-valued force laws can be found in references (Glocker, 2001), (Leine and Nijmeijer, 2004). In this particular case, the normal forces will be determined by the projection of the change of linear and angular momentum in the constrained directions,

$$\boldsymbol{\lambda}_N = \begin{pmatrix} \lambda_{NR} \\ \lambda_{NL} \\ \lambda_{NF} \end{pmatrix} = \mathcal{P}_a\left(\mathbf{q},\,\mathbf{u},\,\dot{\mathbf{u}},\,\boldsymbol{\lambda}_T\right), \quad (12)$$

where $\mathcal{P}_a$ is a set of algebraic equations which are explicitly solved for the normal contact forces. The contact forces can be seen in figure 1. In the absence of impacts the differential inclusion becomes:

$$\mathbf{M}(\mathbf{q},\,t)\,\dot{\mathbf{u}} - \mathbf{h}(\mathbf{q},\,\mathbf{u},\,t) - \mathbf{W}_T(\mathbf{q},\,t)\boldsymbol{\lambda}_T = \mathbf{0}, \quad (13)$$
$$\boldsymbol{\lambda}_N = \mathcal{P}_a\left(\mathbf{q},\,\mathbf{u},\,\dot{\mathbf{u}},\,\boldsymbol{\lambda}_T\right),\ \boldsymbol{\lambda}_N \,\geq\, \mathbf{0}, \quad (14)$$
$$-\gamma_{T_i} \,\in\, \mathcal{N}_{C_2}\left(\lambda_{T_i}\right). \quad (15)$$

The measure-differential inclusion of a general rigid-body mechanical system can be obtained through the integration of the differential inclusion over the atom of impact time and can be stated as:

$$\mathbf{M}\,d\mathbf{u} - \mathbf{h}\,dt - \mathbf{W}_T d\boldsymbol{\Lambda}_T - \mathbf{W}_N d\boldsymbol{\Lambda}_N = \mathbf{0}, \quad (16)$$
$$\xi_{N_i} \in -\mathcal{N}_{C_1}(d\Lambda_{N_i}), \quad (17)$$
$$\xi_{T_i} \in -\mathcal{N}_{C_2}(d\Lambda_{T_i}), \quad (18)$$

where ξ_{T_i} and ξ_{N_i} at the i^{th} contact are given by the following expressions if the Newton impact law is used:

$$\xi_{T_i} = \gamma^+_{T_i} + \epsilon_{T_i}\gamma^-_{T_i}, \quad (19)$$
$$\xi_{N_i} = \gamma^+_{N_i} + \epsilon_{N_i}\gamma^-_{N_i}. \quad (20)$$

A similar projection $\mathcal{P}_v$ can be formulated on the measure-differential level which would make the measure-differential inclusion look like as follows:

$$\mathbf{M}\,d\mathbf{u} - \mathbf{h}(\mathbf{q},\,\mathbf{u},\,t)\,dt - \mathbf{W}_T(\mathbf{q},\,t)\,d\boldsymbol{\Lambda}_T = \mathbf{0}, \quad (21)$$
$$d\boldsymbol{\Lambda}_N = \mathcal{P}_v\left(\mathbf{q},\,\mathbf{u},\,d\mathbf{u},\,d\boldsymbol{\Lambda}_T\right),\ d\boldsymbol{\Lambda}_N \,\geq\, \mathbf{0}, \quad (22)$$
$$-\xi_{T_i} \,\in\, N_{C_2}\left(d\Lambda_{T_i}\right). \quad (23)$$

Here $d\boldsymbol{\Lambda}_T$ is the differential measure of the tangential contact impulse and $d\boldsymbol{\Lambda}_N$ is the differential measure of the normal contact impulse.

3 Treatment of Contact Dynamics and Time-Stepping Integration

The relatively recent developments in the numerical integration of measure-differential inclusions by the time-stepping technique is the main feature of the presented method. The time-stepping integration technique and the concept of MDI originally stems from J. J. Moreau and are deeply rooted in his work (Moreau, 1988). Though in the considered example impacts due to collisions are excluded, the formulation of a measure-differential inclusion dynamics can handle impacts that occur without collisions, such as velocity jumps due to C^0 constraints or the Painlevé phenomenon. The time-stepping integration used here is depicted in figure 2. The integration is an implicit Euler integration with an embedded fix-point iteration to determine the contact forces based on a scheme that is described in (Jean, 1999). The standard time-stepping integration scheme is tailored in this algorithm for the implementation of the shooting method. Since the shooting method requires several hundred evaluations of numerical integrations, it is advisable to keep the complexity of the model not higher than necessary. The main feature is the determination of the normal contact forces/impulses by the projection of the angular and linear momentum changes in the constrained directions of motion, in order to alleviate the computational burden. There are three constrained directions of motion which emanate from the requirement that all of the

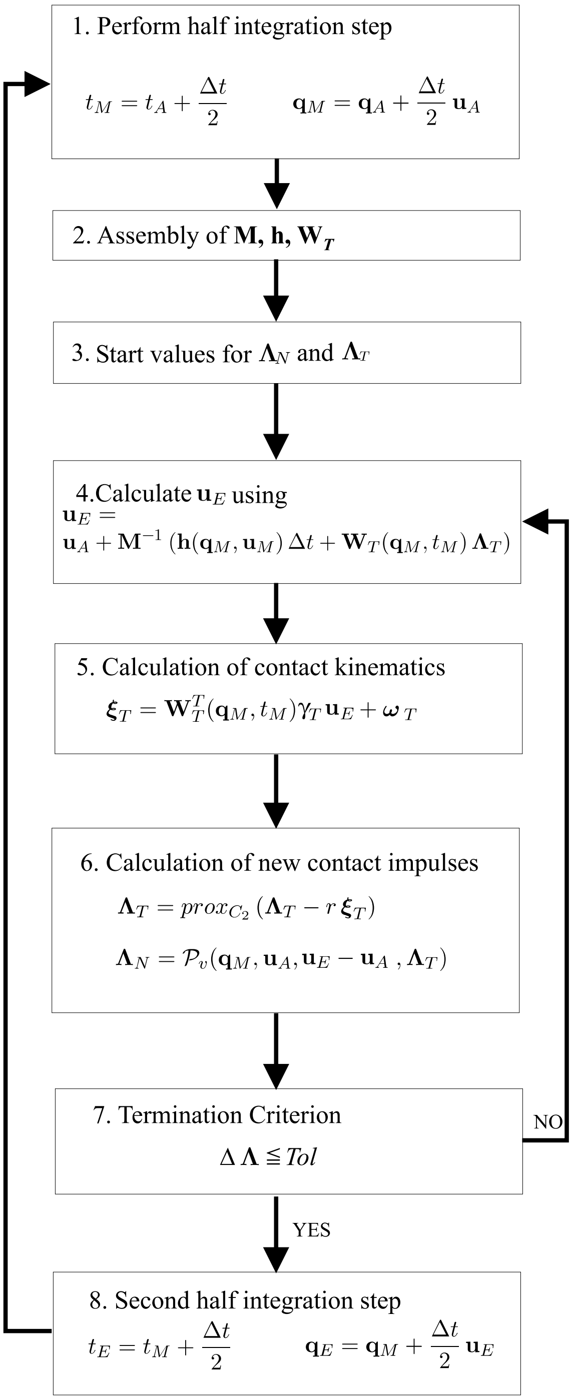

Figure 2. The time-stepping integration scheme applied in the shooting. The tangential contact forces are iteratively solved at each point with exact-regularization along with normal contact forces which are obtained via projection.

contacts should always remain closed. The projection approach reduces the number of DOF to model the system from eight to five.

The replacement of the set-valued inclusions by equalities is the key issue in formulating the iterative scheme to calculate contact forces. The set-valued force laws can be rewritten into equalities by making use of the relation stated in equation (24):

$$\begin{aligned} \boldsymbol{\lambda} &= prox_C(\boldsymbol{\lambda} + r\mathbf{s}), r > 0, \\ &\iff \boldsymbol{\lambda} \in N_C^*(\mathbf{s}), \\ &\iff \mathbf{s} \in N_C(\boldsymbol{\lambda}). \end{aligned} \tag{24}$$

Here $y = prox_C(x)$ denotes the nearest point $y \in C$ to x. The above relation enables the reformulation of the set-valued relation given in (23) as an equality:

$$\mathbf{M}\,d\mathbf{u} - \mathbf{h}(\mathbf{q},\,\mathbf{u},\,t)\,dt - \mathbf{W}_T(\mathbf{q},\,t)\,d\boldsymbol{\Lambda}_T = \mathbf{0}, \tag{25}$$
$$d\boldsymbol{\Lambda}_N = \mathcal{P}_v\left(\mathbf{q},\,\mathbf{u},\,d\mathbf{u},\,d\boldsymbol{\Lambda}_T\right),\ d\boldsymbol{\Lambda}_N \geq \mathbf{0}, \tag{26}$$
$$d\Lambda_{T_i} = prox_{C_2}\left(d\Lambda_{T_i} - r\,\xi_{T_i}\right). \tag{27}$$

At each integration time point, the embedded iterations are stopped when the norm of change of successive normal and tangential contact forces is less than a specified tolerance.

4 Optimization Method

The direct shooting method is used to determine the optimal trajectories, which is a well established method, that bears the advantage that practically no prior information is necessary regarding the adjoint variables compared to the indirect shooting method. The optimization has been performed by the Nelder-Mead simplex method which is a non-derivative based minimization algorithm as described in (Bertsekas, 1999). The goal criteria has been to reach desired end states while minimizing the time. The usage of a non-derivative based optimization method is justified from the point of non-smoothness, non-uniqueness of the solution as well as the difficulty to obtain the (sub) gradient analytically. It is a well-known fact that MPEC do not fulfill in many cases the Mangasarian-Fromowitz constrained Qualifications (MFCQ) as has been presented in references (Luo and Ralph, 1996), (Outrata and Zowe, 1998). The set-valuedness of contact forces, and the linear dependency of friction force directions in this example in particular are closely related to the non-uniqueness of the solution. The non-uniqueness of the solution causes irregularity of the equality constraints leading to the failure of the MFCQ.

The variables for a direct shooting application are chosen as a subset of the final conditions and control variables. The time-stepping integration has been implemented as a C-code, and a MATLAB function is utilized to implement the Nelder-Mead minimization. Given N discretization points, the variable vector $\mathbf{y}$ consists of the discretized moments that drive the

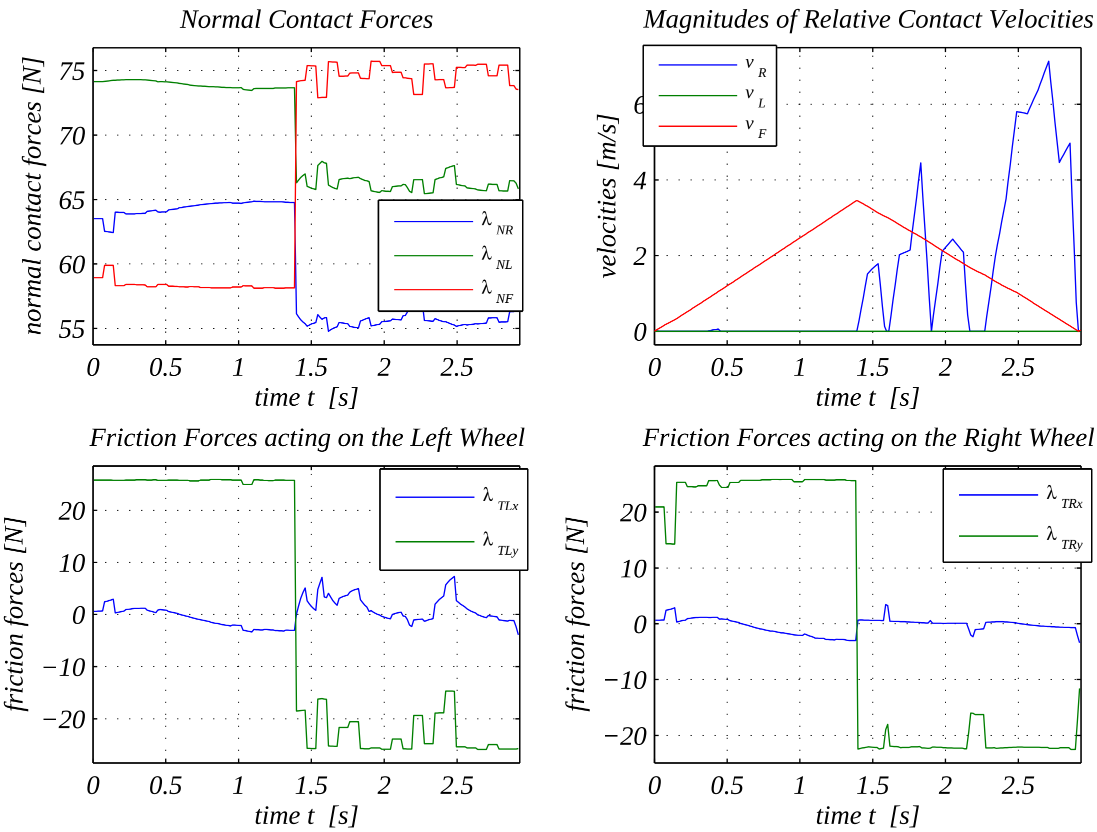

Figure 3. Forward-Drive: Contact Forces and Velocities.

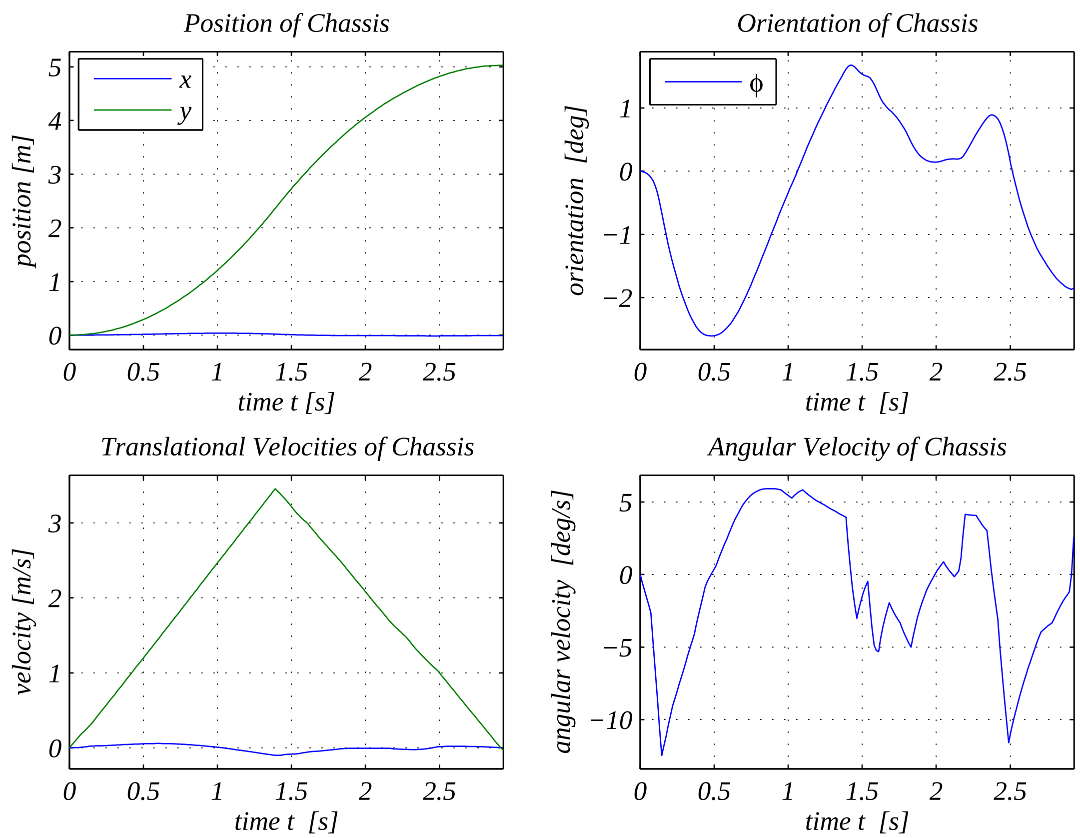

Figure 4. Forward-Drive: Position, Orientation and Velocities of Chassis CM.

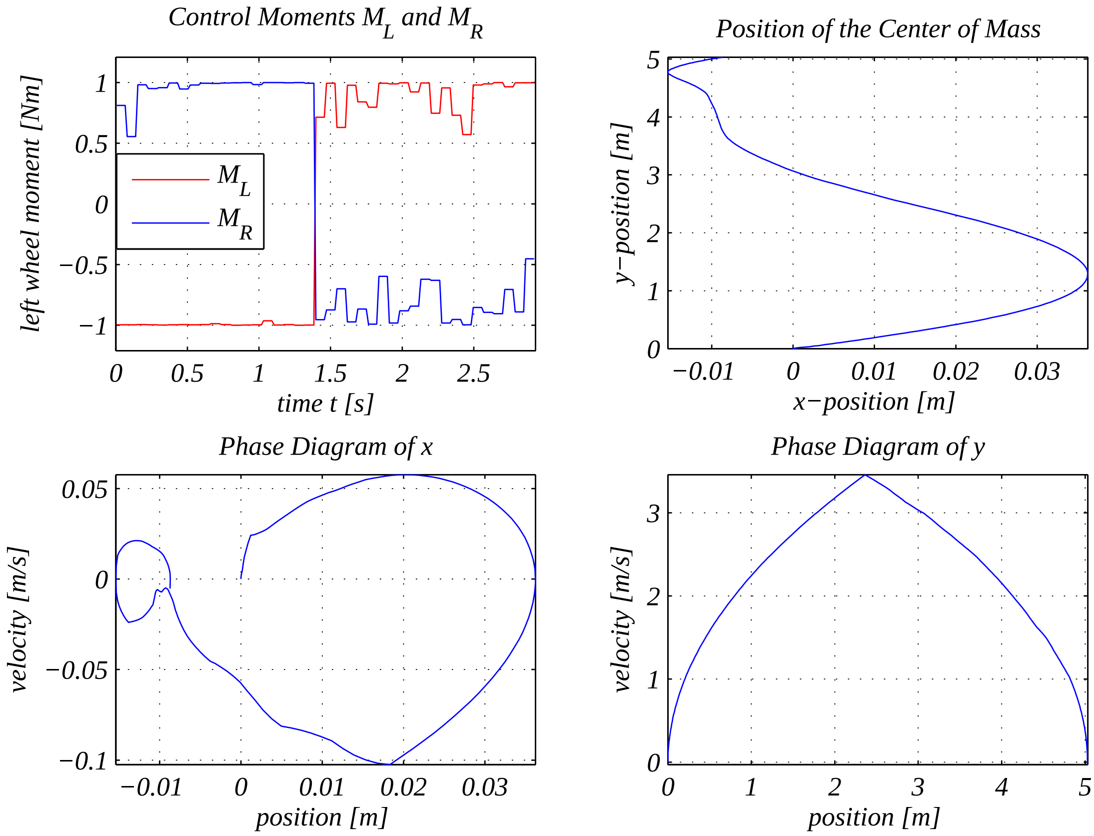

Figure 5. Forward-Drive: Control Torques and Phase Diagrams.

wheels as well as the end time.

$$\mathbf{y} = \{M_R(1), \ldots, M_R(N), M_L(1), \ldots, M_L(N), t_f\}.$$

The deviations on the end state, time and the infeasibility of control moment constraints has been penalized in the penalty function to be minimized. The penalty function consisted of the weighted squared errors of end state deviations, time and squared deviations on the control moments.

The main features of this direct shooting approach can be summarized as follows:

1 Though the underlying system might undergo structure-variant phase changes such as impactive phase transitions and stick-slip transitions multiple shooting is not necessary and a single shooting performs the task.

2 The time-stepping integration takes care of the equalities and inequalities, that emanate from the unilateral and frictional contacts.

3 Knowledge regarding adjoint variables is not necessary at all.

4 The method enables to obtain optimal or suboptimal trajectories to mechanical MPEC in a reduced search space.

5 The sufficiency condition for convergence is a close enough initial trajectory.

In literature there are formulations to convert hybrid system optimal control multiple-shooting problems into a two-point boundary value problem by means of interior point methods so that inequalities are converted into equalities. To the best knowledge of the authors the approach presented here, is the first single shooting scheme proposed for hybrid systems with complementary constraints by making use of exact-regularization. It is also a single shooting method for the solution of bilevel programs arising from mechanical contact problems.

5 Numerical Results

In this section a maneuver will be presented where the robotic system undergoes explicit-phase transitions. In this maneuver the robot is expected to drive straightforward. The center of mass of the robot is not in its geometric center so the distribution of normal forces on the wheels is also affected by this fact. Indeed in real world applications the loading and design of wheeled robots is seldomly symmetric and this trajectory is more probably to occur with oversized motors. Because of the unequal normal contact forces, contrary to the expectations, the robot gets into sliding. In the sequel the trajectories that describe these maneuvers are presented.

The aim is to take the robot from the inital state given by

$$\begin{pmatrix} x_0 \\ y_0 \\ \phi_0 \\ \psi_{L0} \\ \psi_{R0} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} \dot{x}_0 \\ \dot{y}_0 \\ \dot{\phi}_0 \\ \dot{\psi}_{L0} \\ \dot{\psi}_{R0} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

to the final state given by:

$$\begin{pmatrix} x_f \\ y_f \\ \phi_f \\ \psi_{Lf} \\ \psi_{Rf} \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ 0 \\ free \\ free \end{pmatrix}, \quad \begin{pmatrix} \dot{x}_f \\ \dot{y}_f \\ \dot{\phi}_f \\ \dot{\psi}_{Lf} \\ \dot{\psi}_{Rf} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

The control moments of both wheels are limited $\|M_L\| = 1\,Nm$, $\|M_R\| = 1\,Nm$. A number of 200 discretization points are used and the controls are discretized with 40 points each.The optimal final end time is $2.930s$. Figures 3, 4 and 5 depict the optimal trajectories.

The robot tends to drive utilizing full effort straightforward to the end position but due to the unsymmetrical loading it tends to rotate. The unsymmetrical loading can also be seen in the normal contact force trajectories shown in figure 3. This rotation is compensated by the stick-slip transitions of the right-wheel. The robot is able to drive straight in a bang-bang manner thanks to the changes in the degrees of freedom (DOF). In figure 3 the relative contact velocities can be seen. The corners in the phase diagram of y as in figure 5 are typical for a bang-bang type control action. The maneuver can be divided into two main parts. In the first part the system accelerates using full-effort. The second part begins when the system is decelerated. The left wheel contact sticks and fulfills the non-holonomic constraint in both parts of the maneuver, whereas the right wheel begins sliding in the second part of the maneuver. These phase transitions are performed in order to keep the orientation of the chassis straight. In this maneuver the robot has two DOF in the first part and three DOF in the second part of the maneuver. In figure 4 the deviations and the correcting actions can be seen in the trajectories of $\dot{x}$, ϕ and $\dot{\phi}$. The switching to the three-DOF mode can visually be detected also in the trajectories of control moments and contact normal forces as depicted in figures 3 and 5, respectively. In the second part, the right wheel fulfills the rolling condition in small time intervals, which correspond to time intervals where the chassis orientation is nearly straight. The 3-DOF system in the second part of the maneuver also correctis the deviation in x direction, as can be in figures 4 and 5. In the second part of the maneuver, the right wheel gets into sliding as a consequence the sliding friction force is directly related to the normal force λ_{NL} by the friction coefficient. However, the trajectory of the angular velocity Ψ_L is not unique in the sense that it can take any shape as long as the wheel remains in sliding. On the other hand, the right wheel contact always remains in the set-valued region of the contact and it can take depending on the driving moment M_R a range of values because the rolling condition implies sticking of the contact. Contrary, to the freedom in the transmission of friction forces, the trajectory of Ψ_R is strictly prescribed by the kinematics. The possible non-uniqueness in the solutions of the trajectories can be found in this set-valued character of friction contacts.

6 Discussion and Conclusion

A numerical method is presented for the determination of time-(sub)optimal trajectories for mechanical MPEC, which are also typical bilevel programming problems that can handle rigid-body contact problems. The advantages of structure-variant modelling of an differential-drive robot in maneuvers is representative of the advantages that can be obtained by modelling mechanical systems on the basis of MDI. Heuristically, this favorable behaviour can be explained by the ability of a structure-variant system to use a variety of configuration manifolds occasionally enabling the attainment of better goal function values. The method benefits from a sound modelling approach for structure-variant systems based on the time-stepping scheme and the exact-regularization of the complementary character of contacts. The novel feature is that contrary to other shooting schemes for optimization problems with different phases, characterized by different system dynamics, multiple shooting is not necessary and the parameters of event or phase transitions need not to be optimized separately in the optimization. As a consequence, the location and time of phase transitions where the system changes DOF is not prespecified but is determined as an outcome of the optimization. Another novel feature of this approach is that the sequence of different phases does not need to be prespecified either.

The application of the exact-regularization of complementary constraints is not restrained to mechanical structure-variant systems only. The time stepping optimization scheme provides a means to circumvent the difficulties associated with event-driven algorithms that lead to mixed integer programming problems or multiple shooting problems. The globality of the direct shooting method for solving mechanical MPEC is an open issue that has to be clarified because convergence is guaranteed in cases where the initial trajectory has a certain proximity to the optimal solution.

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