3. TIME REVERSE NUMERICAL SIMULATION

3.1 The time reversal method

3.1.1 Introduction
When a phenomenon or event is filmed, the movie can be played back in reverse, giving the impression of going back in time. Fink [14] used the following thought experiment to see if the actual event, instead of the filmed image, can also be reconstructed in reverse chronology.

For instance, can the scattered pieces of an exploded block be used to reconstruct the original structure? Imagine the block to be surrounded by a closed layer, which is crossed by all the particles after the explosion. The position, time of impact and velocity of each particle is measured and recorded when this surface is penetrated. Then, the same speed is applied in opposite direction to each particle at the appropriate time at the corresponding intersection points on the layer. If the slowest particle, i.e. the last one to arrive, is sent back first, and the fastest one last, the initial block should be reconstructed at its origin position. However, this type of experiment only works if the information (such as position, velocity, and time of impact) for each particle is resolved with infinite precision. The explanation of this is the phenomenon of chaos in classical mechanics. According to this theory, a small change in a particle’s initial position can result in a major change in its final position and therefore makes such time reversal experiments impossible.

Another example described by Fink [15] uses a ball which is shot through a fixed array of randomly arranged obstacles. Even in simulation, the ball cannot be played back. Due to minor truncation errors made by the computer, the ball will miss an obstacle sooner or later, which changes the outcome completely. And in a real experiment, it is impossible to start the ball in the exact direction with the appropriate velocity.
Since wave propagation is linear, a minor change in the initial wave results only in a small change of the final wave. Therefore, time reversal can be applied to all wave phenomena, as long as the following restrictions are fulfilled.

### 3.1.2 Basic principles and applications

The time reversal method can be applied to every phenomenon described by equations which contain only second-order derivatives (or, more general, derivatives of even order) with respect to time $t$. Equations of this type are called time-reversal-invariant equations. For each solution $u(r, t)$, there exists a second solution of the form $u(r, -t)$, because the second derivative of $u(r, t)$ is identical to that of $u(r, -t)$. Another restriction for the present approach demands that the medium be non-dissipative. However, it might be possible to reverse the attenuation as well, but this was not taken into consideration in this work.

The time reversal experiments typically consist of two steps. First, a wave field is generated either by a source or by scattering at an obstacle. This wave field $u_s(r, t)$ is measured at different fixed positions (defined by $r$) as a function of time and stored. The duration of this recording is given by $T$. Next, the measurements at each position are reversed in time, which results in the time reversed signals

$$ u_{TR}(r, t) = u_s(r, T-t) \quad (3.1) $$

In the second step, the measurement positions are used as sources where the time reversed signals $u_{TR}$ are applied simultaneously. The resulting waves propagate back through the medium and interfere constructively at the position of the original source.

In the last few years, many time reversal experiments were performed and different applications presented. In 1995, Derode et al. [8] carried out the first surprising experimental demonstration of the reversibility of a multiple scattered acoustic wave. Their setup is immersed in a water tank and includes an acoustic source, a scattering medium comprised of 2000 randomly positioned steel rods and a time reversal mirror (TRM). The TRM consists of an array of receiver and transmitter transducers connected to a device to store signals. In the first step, a very short signal is excited at the point source and propagates through the scattering media. The transmitted waves are recorded at the TRM, using the transducers as receivers. Due to the multiple scattering, these signals are much longer than the initial pulse at the source. In the next step, the recorded displacement histories are reversed in time.
and retransmitted through the medium by the same transducer array operating in the transmission mode. The result is a highly compressed signal with respect to time and space at the position of the initial source.

Further applications of the time reversal method, such as underwater acoustics, medical imaging and pulse-echo detection are described in Refs. [15] and [16]. In the next section, a numerical experiment is executed to demonstrate the applicability of the time reversal method to elastic wave phenomena in solids.

3.1.3 Numerical time reversal experiment

In this example, flexural waves are excited in the center of an aluminum plate (material parameters according to Table A.2 in Appendix A.1). It is described using Mindlin’s theory [36], which includes the influence of rotatory inertia and shear. This higher-order theory must be applied because of the small wavelengths corresponding to the applied frequency range. The plate is lying in the $x$, $y$-plane and the displacement field is described by the transversal displacement $w$ and two rotation angles $\psi_x$ and $\psi_y$. It is assumed that the in-plane displacements are proportional to the plate thickness coordinate $z$ with constant slopes $\psi_x$ and $\psi_y$, and that the out-of-plane displacement $w$ remains independent of $z$. The three equations of motion (principle of linear momentum in $z$ direction and principles of angular momentum in $x$ and $y$ directions) are discretized using second-order central differences (see Chapter 2.2). This yields three explicit equations for $w$, $\psi_x$, and $\psi_y$ at the new time step $n+1$. Although the elimination of the rotation angles would reduce the system of equations to one equation only, this step is not performed here. The reason is that the resulting equation would contain the fourth derivative in time and mixed derivatives with respect to time and space, which cannot be implemented with the finite-difference method.

The displacement $w$ in the center of an aluminum plate (size 0.3 m by 0.3 m, plate thickness $h = 0.002$ m) is excited at 200 kHz. The boundaries of the plate are implemented as being stress-free. 600 cells are used in both directions. Snapshots of the transversal displacements $w$ are shown in Fig. 3.1 at three different times, 20 $\mu$s (a), 40 $\mu$s (b), and 60 $\mu$s (c) after the initiation of the excitation.
Figure 3.1: Out-of-plane displacements of a plate, 20 µs (a), 40 µs (b), and 60 µs (c), after the initiation of the excitation. The square aluminum plate (data see Table A.2 in Appendix A.1) is excited transversely in the center of the plate at 200 kHz. The white dots mark the measurement positions.

During the simulation, the transversal displacement \( w \) and the two rotation angles \( \psi_x \) and \( \psi_y \) are recorded as a function of time at 21 points. These measurement positions are marked in Fig. 3.1 and Fig. 3.2 as white dots. They are distributed in a parallel manner to one side of the plate with a spacing of 5 mm between each other. The principle of this first step is illustrated in Fig. 3.2. The measurements and the excitation in the center of the plate are started at the same time. The differences among the arrival times are clearly visible in the signals at the left, which show the recorded time signals at five different positions.

Figure 3.2: Principle of the first step of a time reversal experiment. During the excitation of the structure (right signal), the displacements are recorded at different positions (white dots) as a function of time (left signals).
In the next step, the measured signals are reversed in time. Then, the same points that were used for the recording are excited with the displacement histories in a time reverse order using an identical simulation model. To obtain the correct wave field, all components of the displacement vectors at the measuring locations must be retransmitted. The reason is that the implemented finite-difference scheme is based on all three displacement components. Therefore, the transversal displacement $w$ and the two rotation angles $\psi_x$ and $\psi_y$ must be recorded and excited in this example. Another important point is that all displacements must be excited simultaneously with the appropriate time delays at the corresponding positions. This step is visualized in Fig. 3.3, where the time reversed recorded signals are shown on the left. Comparing these signals with the measured histories of Fig. 3.2 (left), the principle of last-in first-out becomes obvious. The signals that arrived last during the simulation (in this case, the signals at the top and at the bottom, which correspond to the recording positions furthest away from the source) are played back first, while the fastest signal (middle) is retransmitted last.

During the simulation of the time reversed signals, each measurement point acts as a point source with the corresponding excitation signal. The waves propagate in circular motion from the points and start interfering with each other. In Fig. 3.4 the resulting wave fields are plotted for three different times, 55 $\mu$s (a), 75 $\mu$s (b), and 87 $\mu$s (c) after the playback initiation. The absolute time values are irrelevant and depend only on the used time origin, which is given by the duration $T$ of the recording sequence.
Figure 3.4: Out-of-plane displacements of an aluminum plate (data see Table A.2 in Appendix A.1), 55 µs (a), 75 µs (b), and 87 µs (c), after the initiation of the playback. The recording positions (white dots) are excited with the time-reversed measured signals. The three snapshots show the wave fields before (a), after (c), and at the time (b) with the maximum interference.

If the relation between the grayscale values and the amplitudes is kept constant, one can observe the displacements during the simulation to find the time with the maximum interference, which corresponds to the brightest value in the current representation. Once the time with the maximum amplitude is determined, the displacement field at this time is evaluated in order to find the position of the maximum. The plot in the center of Fig. 3.4 corresponds to the time with the maximum interference.

Fig. 3.5 shows a three-dimensional zoom of the center area of the plate at the time with maximum amplitude. The position with the maximum displacement is found by analyzing Fig. 3.5 and has the coordinates 0.15 m and 0.15 m. This corresponds to the center of the plate, where the original pulse was initiated. The amplitude increase at the maximum interference time can be clearly distinguished from the amplitude field at other times.

The numerical example performed confirmed that the time reversal method can also be applied to wave propagation experiments, at least in simulations. As long as the displacements and the rotation angles are recorded and retransmitted at some positions, the origin of a wave field (in this case the point source in the center of the plate) can be determined, even if it is far away. This is an important finding for the application of the time reversal method for detecting defects which is explained in the next section and in [28] by the author.
3.2 Detection of defects using the time reversal method

The experiments of Derode et al. [8] with steel rods in water, and the numerical example of the plate described in the previous chapter show that the time reversal method can be used to detect the origin of a recorded wave field. In both cases, the waves are generated by a point source and recorded at different locations. Then, the time reversed signals are retransmitted and interfere to the maximum extent at the position of the source. Instead of using the method to determine the original source, it can also be applied to a defective structure in order to determine the presence of a defect and its exact position. This idea was developed and implemented by the author who named it time reverse numerical simulation (TRNS) method. This new approach is explained below.

Imagine a guided wave that propagates along a structure and interacts with a present defect. The result is a scattered field which contains a part of the energy of the initial waves. The scattered waves are then recorded at certain measuring positions. This step is outlined in Fig. 3.6 for a tube with an axial notch used as an example. The axisymmetric waves initiated at the left end of the tube interact with
the notch, which generates the scattered field as indicated. These reflected waves are recorded at the measurement points distributed equally around the circumference at the outer surface at the left end of the tube.

Even though the waves are excited at the beginning of the tube, the notch can be viewed as the origin of the scattered field. The defect acts as a source where the scattered waves are generated during the interaction with the initial wave pulse. The fact that the length of the notch in Fig. 3.6 is in the range of the wavelength of the excited waves is only coincidence.

According to the previous plate example, the time reversed displacement measurements are used to determine their origin. If the recorded displacement histories are reversed in time and used as excitation signals at the appropriate measuring positions, the waves travel back through the structure and interfere with each other.

In order to obtain the correct location of the maximum amplitude, the structure used for playing back the time reversed signals must be identical with respect to material parameters, boundary conditions, and geometry, and the measuring and retransmission positions must be the same.

The maximum amplitude which results from the constructive interference is obtained at the position of the source, which is exactly the beginning of the notch. Fig. 3.7 shows a diagram of the played back waves at the time with maximum interference. The position of the notch is indicated by the dashed rectangle.
As noted previously, the time reversal method only works properly, if all displacement components are recorded at the measuring positions. In the case of cylindrical structures, these are the radial, tangential, and axial displacements $u_r$, $u_\phi$, and $u_z$. As shown with different simulations, it is sufficient to record the three displacements only on the outer surface of the structures, as long as the wavelength is large compared to the thickness of the tube. The reason is that for this frequency range, the wave propagation directions are mainly axial and circumferential.

**Figure 3.7:** Diagram of the played back waves at the time with the maximum interference. All three displacement histories are time reversed and played back in the corresponding locations on the outer surface of the tube. The highest amplitude occurs at the origin of the scattered field which is the beginning of the defect. The position where the notch was located during the first step is indicated by the dashed rectangle.

The recording of the three displacement components in the first step can be done experimentally, using a special laser interferometer which is described in Chapter 4.1. However, for the playback part, imposing precise displacements in three directions at the measuring positions of the structure is difficult in an experiment, especially when dealing with small samples.

Another challenge is the detection of the amplitude increase due to the interference within the structure. If the playback is done experimentally, the tube must be scanned in order to find the maximum amplitude. This procedure is performed by Ing et al. [26] who use a laser interferometer to find the origin of time reversed lamb waves in thin plates. Since the main goal of this thesis is the development of a new method for the fast NDT of large structures, the aim is to eliminate such
steps. For this reason, and also to avoid the complex excitation of the displace-
ments and the scanning of the sample, the playback part is replaced by a numerical
simulation. This enables also the visualization of the position of the maximum
amplitude. In this case, imposing the time reversed displacement histories is easy.
As long as the simulated structure shares the same geometrical and material param-
eters and the same boundary conditions as the sample used in the experiment, the
reverse wave propagation can be calculated numerically. To obtain accurate
results, the physics of the wave propagation phenomenon must be described appro-
priately by the simulation. To fulfill this requirement, a three-dimensional code in
cylindrical coordinates with a high accuracy must be used. The finite-difference
codes described in Chapter 2. meet these demands.

One important characteristic of the time reversal method is that even though no
defect is present in the simulated structure used in the playback step, the waves
interfere constructively, and the maximum displacement or stress takes place at the
position where the defect is located in the sample used for the recording. This is
one of the key features of the TRNS method. It means that measured signals
obtained in an experiment performed on a defective structure can be time reversed
and used as input signals in a notch-free simulation to determine the position of a
defect within a large structure. A summary of the major steps of the TRNS method
is provided below:

- In a physical experiment, guided waves are excited in a defective tube.
  Compared to other NDT methods using guided waves, the initiation of the
  waves is not a crucial step in this approach. It is not necessary to excite one
  specific mode. Basically, any method that generates scattered waves at the
defect is applicable.

- The amplitudes of the scattered waves that are generated at the defect are
  measured as a function of time at different positions. All three displace-
  ment components must be recorded, which is done by using a three-dimen-
  sional laser interferometer. It is sufficient to measure points on the surface
  of the tube only.

- A structure with the same geometrical and material parameters is imple-
  mented in the simulation code. This structure is free of defects and must
  have the same boundary conditions as the sample used in the experiment.

- The recorded signals are time reversed and used as excitation signals in the
  simulation. Therefore, the points in the simulated structure that correspond
to the measuring positions are driven simultaneously in all three directions, with the corresponding signals and the appropriate time delays.

• The waves start to propagate back through the structure and begin to interfere with each other. During the playback simulation the displacement amplitudes are observed in order to determine the time and position of the maximum interference.

• From this location in the simulated structure, the axial and circumferential coordinates of the defect within the defective tube are calculated by using the corresponding discretization parameters. Since the time signals are not analyzed directly, it is neither necessary to determine any time differences nor to identify the scattered wave modes or their group velocities.

Until now, the time reversal method was either used for strictly theoretical examples or as a tool for investigations where both steps, recording and retransmitting, are performed experimentally on the same sample. In contrast to the applications found in literature, the TRNS method at hand combines experiments and numerical simulations. The result of this new combination is an efficient and accurate method for the detection of defects in real structures. Since scanning the sample is eliminated by using simulations, the present approach can also be applied to tubes with insulation or limited access. The experiments performed and results achieved are presented and discussed in Chapter 4.

3.3 Verification of the method using simulated data

To validate the TRNS method, the whole procedure is simulated. Accordingly, the first two steps are performed in a simulation as well instead of an experiment. The defect is approximated by a notch implemented into the FDM code as described in Chapter 2.2.4. Then, this simulation of the defective structure is excited, and the scattered field is recorded at the measuring points. The next steps are performed following the standard procedure of the TRNS method.

This verification allows the user to check the concept and applicability of the TRNS method to large structures and to test the accuracy of the introduced approach. Thanks to the simulation of all steps, the following uncertainties and errors encountered during the experiment can be eliminated:
• The material parameters and the geometry of the defective structure used for the generation of the scattered field are known in detail due to their exact definition in the simulation code. This prior knowledge eliminates the determination and measurement of these values in the sample to be tested.

• The recording positions are identical to the retransmission points, and the three displacement directions match the radial, tangential, and axial components. Errors resulting from the three-dimensional laser interferometer and positioning deviations are excluded.

• The excitation of purely axisymmetric wave modes is possible in the simulation. Although this is not necessary, it simplifies the interpretation of the recorded time histories. Another advantage is that the implemented structure is homogeneous, geometrically axisymmetric, completely straight and of even wall thickness. This assures that the structures used to record and play back the waves are identical except for the notch.

• Since no measuring equipment, excitation transducers, interferometer and optical devices are used for the verification, the recordings are free of noise or offsets, and the piezo element has no influence.

An aluminum tube (mass density \( \rho = 2700 \text{ kg/m}^3 \), Young’s modulus \( E = 6.9 \cdot 10^{10} \text{ N/m}^2 \), Poisson’s ratio \( \nu = 0.34 \)) with a wall thickness of \( h = 0.002 \text{ m} \), with a radius of midplane \( R = 0.015 \text{ m} \), and a length of \( l = 2 \text{ m} \), is used for the validation. A part-through circumferential notch is implemented at a distance of 0.8 m from the excitation location at the left end. The notch starts at the outside of the structure, is 1 mm deep and 1 mm wide. The circumferential extent is 9.5 mm which corresponds to approximately 34°. 4, 106, and 2000 cells are used in radial, circumferential, and axial directions, respectively. The left end of the tube is excited axisymmetrically in axial direction across the wall thickness with 10 cycles of a sine wave at a frequency of 200 kHz. According to the dispersion diagram in Fig. 1.1, two wave modes [L(0,1) and L(0,2)] are expected for the applied axisymmetric excitation at the chosen frequency. The driving function is multiplied by a Hanning window before being applied. This is equivalent to the experiment described in Chapter 4.5.1.

Four snapshots of the axial displacement at different times are shown in Fig. 3.8. Since the excitation is above the first cutoff frequency, the L(0,2) mode exists and is clearly visible. Of course the L(0,1) mode also occurs but is barely
visible in this representation of the axial displacement. This is due to the ratio between the radial and axial displacements of the two modes. For the geometry and frequency in this example, the axial displacements of L(0,1) at the outer surface are much smaller than those of L(0,2).

Figure 3.8: Interaction with a circumferential part-through notch that is 0.8 m away from the left end of an aluminum tube (see Table A.2 in Appendix A.1) of 2 m length. The axial displacements at the outer surface are shown at 130 $\mu$s (a), 180 $\mu$s (b), 231 $\mu$s (c), and 300 $\mu$s (d) after the axial excitation at 200 kHz at the left end. The tube is plotted in a developed view, with the axial length on the horizontal, and the circumference on the vertical axis. The circumference is enlarged for better visualization.

To display the entire tube surface, a developed view is chosen with the axial and circumferential extent on the horizontal and vertical axis, respectively. For better visualization, the circumference, which is about 0.1 m for the present tube, is enlarged relative to the length of 2 m. The amplitudes are represented as different gray values with a constant range for all four snapshots.

During the simulation, radial, tangential, and axial displacements are recorded in 106 points distributed equally around the circumference on the outer surface at the left end. Fig. 3.9 shows the recorded time histories in one measuring location. The amplitudes are normalized with respect to the maximum excitation in axial direction. The amplitude ranges of the three displacements are different.
Figure 3.9: Recorded time histories of the radial, tangential, and axial displacements in one of the measuring points at the outer surface at the left front side of the tube. The amplitudes are normalized with respect to the maximum applied excitation in axial direction.

The time histories are recorded for a duration of 900 $\mu$s. The wave packet in the axial displacement at around 800 $\mu$s corresponds to the first reflection of the $L(0,2)$ mode from the right end. Between the excitation and the first reflection, many different wave packets exist, and the time signals are difficult to interpret.

Fig. 3.10 shows the group velocity dispersion curves for the simulated tube for circumferential orders $n = 0, 1, \ldots, 10$ and mode numbers $m = 0, 1, \ldots, 10$. Since $L(0,2)$ is the fastest mode in the selected frequency range, all waves between the excitation and the first reflection from the end must correspond to reflections from the defect and therefore have the same propagation direction.
Figure 3.10: Group velocity dispersion curves for the tested aluminum tube (data see Table A.2 in Appendix A.1) for longitudinal L(0,m), torsional T(0,m), and flexural F(n,m) modes. The curves for mode numbers m=0,1,...,10 and circumferential order n=0,1,...,10 are plotted. The unlabeled lines correspond to flexural modes.

Since the excitation is perfectly axisymmetric, the first non-axisymmetric signals must be reflections from the notch. They are most visible in the tangential displacement, which is zero until around 300 µs. After this first reflection from the notch, many more wave packets arrive at the measurement positions. Since the scattered field is generated at the notch, which is a quasi point source, it changes from position to position and cannot be described by plane waves near the defect. Even far away from the notch, only the first wave packets in the time histories can be assigned to specific modes. The reflections at later times can no longer be classified since the waves modes are not separated. Therefore, the determination of their group velocities from dispersion diagrams becomes very ambiguous.

As described in Chapter 2.2, the stress components are also calculated explicitly in the present FDM code. The disadvantage of this approach in terms of memory requirements and computational time turns out to be an advantage for the TRNS method. The interaction of the excited waves with a defect are most visible in the stress components. As examples, the six stress components are plotted in Fig. 3.11 and Fig. 3.12 at four different times during the generation step of the scattered field. The same times are chosen here that were used for the snapshots of the axial displacements in Fig. 3.8. The notch is plotted as a white line, and the amplitudes are normalized with respect to the highest stress that occurs during the simulation. For the current excitation in axial direction, these are the axial normal stresses $\sigma_{zz}$. 
Figure 3.11: Snapshots of the stress components below the outer surface are shown 130 µs (top) and 180 µs (bottom) after the axial excitation at 200 kHz at the left end.
**Figure 3.12:** Snapshots of the stress components below the outer surface are shown 231 µs (top) and 300 µs (bottom) after the axial excitation at 200 kHz at the left end.
The scattered field is much more visible in the stress components than in the axial displacements of Fig. 3.8. Even the L(0,1) mode is apparent in the shear stress component $\sigma_{rz}$ in Fig. 3.11 (top). The reason for this is that the range of this component is small enough before the L(0,2) mode reaches the notch. At 180 $\mu$s after the excitation (Fig. 3.11 bottom), the L(0,2) mode interacts with the notch. The beginning of the scattered field at the defect is most visible in the shear components, which are very small or zero before the interaction with the notch for the chosen axisymmetric excitation. After the L(0,2) packet has crossed the notch, a part of the scattered field propagates back towards the left end. The other part travels in the same direction as the initial waves (Fig. 3.12 top) but at a lower speed than the original L(0,2) mode. The excited L(0,2) mode is always faster than all other wave modes generated at the notch, which is visible in the dispersion diagram in Fig. 3.10. Since the scattered field consists of a superposition of many wave modes, it changes with time and space. The different group velocities become apparent by comparing the snapshots of Fig. 3.12, top and bottom.

In the next step, the notch is removed from the previously used structure, and another simulation is performed. For excitation, the time reversed signals recorded at the 106 positions at the left end, are applied simultaneously at the corresponding points. In the example at hand, the first 750 $\mu$s of the displacement histories are played back, excluding the reflections from the right end. As a result, the scattered waves travel back along the tube. Snapshots taken at four different times during the playback simulations are depicted in Fig. 3.13 and Fig. 3.14. Again, the absolute times are irrelevant and depend only on the duration of the recording and the used time origin. At the top and bottom of Fig. 3.13, the snapshots are shown at 456 $\mu$s and 531 $\mu$s. The various wave packets and their different group velocities can be seen. The snapshot at the top of Fig. 3.14 depicts the stress components at the time when the maximum amplitudes occur. The wave field is focused on one location, which corresponds exactly to the position of the defect at 0.8 m from the left end. The position is clearly visible in all stress components. At later times, the waves start to separate again (see Fig. 3.14, bottom). The amplitudes of the stress components are normalized with respect to the maximum value that occurs during the TRNS. This is the axial normal stress $\sigma_{zz}$ at 573 $\mu$s. The comparison of the stress amplitudes during the playback simulation shows an intense increase of up to 100% when the waves reach the location of the defect. This enables an accurate determination of the axial and circumferential position of the notch.
Figure 3.13: Snapshots of the stress components 456 µs (top) and 531 µs (bottom) after the start of the playback of the recorded displacement histories.
Figure 3.14: Snapshots of the stress components 573 µs (top) and 648 µs (bottom) after the start of the playback. The top picture shows the time when the maximum amplitude occurs. The position of the notch is clearly visible and matches the exact location.
Since the scattered field is generated in a simulation, the recorded time signals and the TRNS method in its entirety are of high quality. Accordingly, the position of the defect can also be detected by observing the displacements. Fig. 3.15 (top) shows the developed view of the structure with the defect that was used for the generation of the scattered field. The bottom picture displays the axial displacement field at the time with maximum interference. The position of the notch is clearly visible and matches the exact location.

However, in real experiments, the positions can be determined from the displacement signals only with difficulty, which is why the stress components are observed there.

It is important to play back only direct reflections from the defect. If reflections of the transmitted field from the end of the structure are retransmitted as well, the defect cannot be located correctly. Assume, for this example, that the displacements are recorded for a duration of 2000 μs. These histories also include the first reflection of the L(0,2) mode from the right end (after 800 μs) and the fastest transmitted wave packets which were generated at the notch and reflected at the right end. Since these reflections travel along the whole structure twice before reaching the recording positions, they arrive much later. In the playback part, this is compensated for by exciting them first. The played back reflected waves from the end are also reflected at the right end in the time reverse simulation and travel back towards the recording or excitation positions. On their way back, they cross the direct reflections from the defect that are retransmitted later at the left end. Even if the time delays are appropriate, the two fields do not interfere construc-
atively at the position of the defect. This is due to the fact that the reflections from the end of the structure pass the defect twice before being recorded. When played back, they cannot interact with the defect on their first path because no defect is present in the sample used for the time reversed simulation. This changes the path of the waves so that they can no longer interfere correctly. Therefore, it is important to only play back signals that interacted just once with the defect. For this example, this means that only the first 750 $\mu$s of the recorded displacements are time reversed and retransmitted.

### 3.4 Conclusions

Since elastic wave propagation is described by time-reversal-invariant equations, wave phenomena can be reversed in time and played back. This is demonstrated for the example of flexural waves in a plate, where the recorded wave field is time reversed and played back. The retransmitted waves interfere and reach a maximum amplitude at the position of their origin, which can be either a source or a notch. This is the key element of the developed time reverse numerical simulation (TRNS) method. Structural waves are generated in a defective sample, and the scattered field that results from the interaction with the defect, is recorded at several positions. Then, these signals are time reversed and used as excitation signals in a numerical simulation of the same structure, but without a defect. The played back waves refocus at the position of the defect as long as all three displacement components are excited simultaneously. It is found that it is sufficient to only record the displacements on the surface of the structure. The advantage of using a simulation for the playback part is that the position and time of the maximum amplitude can be determined by observing the displacement and stress components during the simulation. Hereby, the time-consuming scanning of the structure, which would require overall access and the removal of any possible insulations, is eliminated. The simulation of a complete experiment allows the verification of the TRNS method and its applicability to the NDT of large structures.